# Long-Term Unemployment and Technical Progress in an Open Growth-Matching Model

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**Abstract**

How does technical progress affect long-term unemployment in a small open economy? This relationship is evaluated in an open growth model extended by a Pissarides-style labor market approach. In the general equilibrium model, the labor market is characterized by two types of unemployed workers. International capital mobility represents the efficient intertemporal allocation of world resources. Depending on a growth condition, I show that an increase in the growth rate implies a favorable capitalization effect respectively an unfavorable creative destruction effect. Secondary effects in form of a stigmatization and a human capital depreciation effect are also generated that influence the unemployment duration and the fraction of long-term unemployment negatively. The higher the growth rates are, the larger the fraction of long-term unemployment. Due to the openness of economies, new technologies are easily spread into various countries, which increase the fraction long-term unemployed workers.

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# 1 Introduction

Data on long-term unemployment[1] show huge increases in the level and the growth rates of long-term unemployment for various industrialized countries.[2] Beside high levels of benefit payments, bearing some accountability for the rising fractions of long-term unemployment,[3] I will argue in this paper that the fast pace of new technologies, which took place for nearly two decades until the end of the 1990s, had not only positive effects on employment but strong negative implications for *long-term* unemployed workers. Workers who are out of work for a relative long period of time will have an even harder time to handle and to adjust onto new technologies. The fast spread of new technologies was possible because markets are much more integrated nowadays than a couple of decades ago. Putting both aspects—the impinge of new technologies upon long-term unemployed workers and the fast spread of new technologies—together, I will ask what is the effect of high growth on long-term unemployed workers when countries have easy access to technologies? That is, how were especially long-term unemployed workers affected by the high speed of new technologies we saw since the mid 1980s?

I will discuss the linkage of growth on long-term unemployed workers in an open growth model extended by a Pissarides-style labor market approach. Since no attempt is made to explain growth explicitly, the neoclassical growth model with exogenous technical progress seems to be appropriate.[4] To analyze structural changes in the labor market generated by the fast introduction of new technologies, the Pissarides-style matching approach is chosen, because the flow approach is able to describe the induced structural changes.

My theoretical framework merges different strands of the literature. First is the literature[5] on growth and employment describing *capitalization* and *creative destruction* effects. If growth comes through creative destruction (Aghion and Howitt, 1992), the flow of workers into the pool of unemployed and thus the equilibrium unemployment rate is increasing in the growth rate of the economy. On the other hand, a higher equilibrium growth rate induces higher future revenues and thus rising vacancies that lead to more employment (capitalization effect, see Bean and Pissarides, 1993). Both effects appear in my paper simultaneously. I will show that an increase in technical progress leads via the net creation of jobs to the capitalization effect, as a growth condition has been satisfied: If the growth rate of the small open economy is higher than an average of the interest rate and the job separation rate, rising employment and shrinking unemployment is implied. However, violating the growth condition, the negative Schumpeterian creative destruction effect is caused by the net destruction of jobs and average unemployment will increase. Both effects are simultaneously captured within my simple open growth-matching model. That is, the increase in technical progress implies the favorable capitalization effect as well as an unfavorable creative destruction effect, which depends on satisfying a growth condition, for unemployed workers.[6]

Beside these primary implications, i.e. the capitalization and creative destruction effect acting directly but in opposite directions on unemployment, secondary effects called stigmatization and human capital depreciation effect will arise as well. Both effects will have a negative influence on the duration of average unemployment and on the fraction of long-term unemployment. The *stigmatization effect* shows the relation of technical progress on the duration of unemployment. Increasing technological progress implies a higher duration of unemployment, since faster technical progress decreases the matching probability for long-term unemployed and less of them will leave the unemployment pool. That is, with higher productivity growth, the duration

[1] Long-term unemployment is defined as the *number* of unemployed workers who are out of work for more than 12 months.

[2] See Figure 7 in the Appendix.

[3] High benefit payments are ususally made accountable for the huge increase in unemployment and the duration of unemployment. See for example Ljungqvist, Sargent (1998, 2005) and Layard, Nickell, Jackman (1991).

[4] To explain growth endogenously see for example Romer (1986, 1990), Aghion, Howitt (1992).

[5] See for example Mortensen, Pissarides (1998, 1994), Merz (1999, 1995), Postel-Vinay (1998), Bean, Pissarides (1993).

[6] For both effects, acting only on employment, see also Aghion, Howitt (1994, p. 477) and Postel-Vinay (1998, p. 1101).

for long-term unemployed will increase and they will be stigmatized by higher durations.[7] In the model with short- and long-term unemployed workers, I show furthermore that increasing technical progress will cause a *human capital depreciation effect* for long-term unemployed workers. Being out of work for a long period of time implies that the latter group does not have the know-how and the abilities necessary to handle the latest production technologies. Firms demand only short-term unemployed for their vacancies endowed with the latest technologies, which induces an increasing fraction of long-term unemployed workers. Due to human capital depreciation, the fraction of long-term unemployed will rise even further as the speed of technical progress becomes faster.

The latter link between the skills of unemployed workers and technological progress is also discussed in recent studies showing the negative implications of skill-biased technological shocks on (long-term) unemployment.[8] Most of these studies conclude that skill-biased technological shocks have a greater negative impact on long-term than on short-term unemployed workers. As new production technologies are implemented very rapidly, the human capital of long-term unemployed depreciates stronger than that of short-term unemployed inducing the strong adverse effect.

The literature explains usually boosting unemployment in matching models for closed economies.[9] Since the easy access to new technologies contributes a lot to their fast distribution, this paper will focus on an open economy to capture the effect of nearly no boundaries on trade, we see in the industrialized countries nowadays.[10] That is, my paper expands the literature also in that respect by discussing the relations between capital accumulation, technical progress, the labor market and capital as well as goods market integration in an open economy. It is well-known that in small open economies, with an indefinitely-lived representative household, a long-run steady-state equilibrium does not exist when the subjective discount factor is different to the world interest rate.[11] In this paper, I will show that the existence of a steady-state in the small open economy can be reestablished by assuming that households consume and save a constant fraction of their income. In that case and even if the small country is a net debtor, its capital stock will not be consumed and, therefore, the country will not vanish in the long-run. Since the country does exist in the long-run, I will further show, as a minor result, that it can have a sustained current-account surplus or deficit in the steady state. That is, the model does not support any notion of a long-run well balanced trade; instead a long-run current-account surplus or deficit might be sustained in the steady-state.[12] With respect to high growth rates, I show furthermore that the current-account surplus for a net creditor country will increase even more as growth rises.

To summarize, the paper tries to explain the influence of *high* growth rates of technical progress especially on *long-term* unemployed workers in an *open* growth-matching model. The model is developed in the next section. Section 3 analyses the steady-state solution and the stability of the model. Thereafter, economic implications are discussed in section 4 and Section 5 concludes.

[7] See also Blanchard, Diamond (1994).

[8] See for example Coles, Masters (2000), Marimon, Zilibotti (1999), Mortensen, Pissarides (1999), Ljungqvist, Sargent (1998, 2005).

[9] As an exception, see for example Feve/Langot (1996). They integrate the search labor market approach into a small open real business cycle model and estimate the implications for the French economy.

[10] This is particularly true for the countries of the EU and for the NAFTA countries.

[11] See also Obstfeld (1990, p. 46): "In a deterministic setting with infinitely-lived households, constant time-preference rates fail to produce a steady-state in which all households have positive net worth and consumption. Translated to a world of small open economies, the result implies either that the long-run global distribution of wealth is indeterminate or the economy with the lowest time-preference rate eventually comes to own all the world's outside wealth."

[12] This model might therefore reflect the U.S. current-account deficit over the past decades.

# 2 The Economy

## 2.1 The Labor Market

Aggregate labor endowment of households is constant and denoted by $L = \overline{L}$. At any time labor is either employed or unemployed; the employed workers are denoted as $E$ and the unemployed as $U$. Thus, the labor force is represented by

$$\overline{L} = E + U. \tag{1}$$

The labor market is characterized by search frictions with firms looking for jobless workers filling vacancies and unemployed searching for a job. Both sides of the market have incomplete information about the opposite market side. The level of search activities is represented by the number of vacancies $V$, the number of unemployed $U$ and the number of matches $M$ formed at any point in time. If no frictions were present, laid-off workers would find immediately new jobs and equilibrium unemployment would not exist.

*Matching and Technical Progress*

Since newly created vacancies are endowed with the most recent technology, the number of matches is also determined by the rate of technological progress $\widehat{\lambda}$ that represents the diffusion of technological know-how. If an economy has a high rate of technological progress, only few unemployed workers can fill the vacancies and the number of matches is relatively low. In order to describe this, it is assumed that technological knowledge of the unemployed does not grow with the same rate as technological progress. The underlying matching technology is defined as

$$M = m(V, U; \widehat{\lambda}) = V^{1-\beta} U^{\beta} \widehat{\lambda}^{-1}, \quad 0 < \beta < 1 \tag{2}$$

with $\beta$ as the search intensity of the unemployed workers. The matching function is assumed to be homogeneous of degree one. This implies that the indicator for labor market tightness is denoted by the ratio of vacancies to unemployed $\theta := V/U$ and

$$p(\theta; \widehat{\lambda}) := M/U = m(V/U, 1; \widehat{\lambda}), \quad p_{\hat{\lambda}} < 0 < p_{\theta} \tag{3}$$

is the matching-probability for the searchers and

$$q(\theta; \widehat{\lambda}) := M/V = m(1, U/V; \widehat{\lambda}), \quad q_{\theta}, q_{\hat{\lambda}} < 0, \tag{4}$$

is the probability of filling the firm's vacancies. Both probabilities depend on labor market tightness and reflect the externalities each trading partner faces. If the number of jobless workers increases, the matching-probability for the average unemployed will decrease and simultaneously the probability of filling vacancies will increase. Introducing the growth rate of technological progress into the matching function implies that it will become more difficult for an average searcher to leave via a job-match the unemployment pool. On the other hand, since the vacancies are endowed with the latest technology, the probability of filling a firm's vacancy will decrease as technical progress increases.

*Human Capital Depreciation and Long-Term Unemployed*

Due to constant returns of scale, the average duration in unemployment is defined as

$$\rho(\theta; \widehat{\lambda}) := U/M = m(V/U, 1; \widehat{\lambda})^{-1}, \quad \rho_{\theta} < 0 < \rho_{\hat{\lambda}} \tag{5}$$

and it goes up when the labor market becomes tighter which is characterized by increasing unemployment for given vacancies. Additionally, as technical progress raises, less unemployed are matched to jobs, and unemployment duration increases as well.

Furthermore, two types of jobless workers are distinguished: short-term and long-term unemployed, $U^S$ respectively $U^L$, and the heterogeneous unemployment pool is defined as

$$\begin{aligned} U &= U^S + U^L \\ U &= [1-\phi(\rho;\widehat{\lambda})]U + \phi(\rho;\widehat{\lambda})U, \quad 0<\phi<1, \ \ \phi_\rho, \phi_{\widehat{\lambda}} > 0, \end{aligned} \tag{6}$$

with $\phi(\rho;\widehat{\lambda})U$ as the long-term unemployed. The long-term jobless workers show significant different search behavior than short-term unemployed. They are looking for new jobs with less search intensity and, due to the long unemployment duration, they are demoralized and discouraged.[13] During their jobless time their human capital is exposed to large depreciation losses and, since they are not trained and do not accumulate any additional knowledge, i.e. without allocating any resources to the long-term unemployed, they are not able to handle the latest production technologies. Therefore, the number of long-term jobless workers depends positively on the unemployment duration $\rho$ and positively on the rate of technical progress $\widehat{\lambda}$.

*Wage Determination*

Due to matching-frictions, trading partners have monopoly power and successful matching yields additional profits which are shared between firms and workers. The division of profits can be modelled by a Nash bargaining approach or simply by sharing the marginal product of labor, whereby the sharing proportions are determined by the bargaining power of searchers and firms.[14] This sharing rule determines the profit proportion newly hired workers get and, because all job-worker pairs are equally productive, the wage rate results as a constant fraction of the marginal product of labor[15]

$$w = \omega F_E(k), \quad 0<\omega<1. \tag{7}$$

$\omega$ denotes the sharing proportion and represents the bargaining power of searching workers.

## 2.2 The Wealth Market

Each firm uses three inputs capital $K$, labor $L$, imports $Z$ and the current state of technological progress $\lambda := \lambda_0 e^{\hat{\lambda}t}$ to produce a homogenous good $X$. Imports are intermediate goods needed to produce the output which is described by a Cobb-Douglas-function:

$$X = F(K, Z, \lambda E) := K^\alpha Z^\gamma [\lambda E]^\varepsilon \tag{8}$$

with $\alpha$ as the production elasticity of capital $[0<\alpha<1]$, $\varepsilon$ as the production elasticity of labor in efficiency units $[0<\varepsilon<1]$ and $\gamma$ as the elasticity of imports $[0<\gamma<1]$. The production function has constant returns to scale and, using $\varepsilon := 1-\alpha-\gamma$, it can be rewritten in efficiency units as

$$x = z^\gamma k^\alpha, \tag{9}$$

with $z$ as imports in efficiency units, $z := Z/\lambda E$,

$$k := \frac{K}{\lambda E} \tag{10}$$

$0<\alpha+\gamma<1$.

[13] See also Layard, Nickell, Jackman (1991).

[14] See also Nickell (1999) and Zanchi (2000) for a recent discussion of the wage determination in search models.

[15] See also Saint-Paul (1996, p. 138) who assumes that “the output of any worker is equally split between the firm and the worker.”

Demand decisions for the representative firm concern changes in real capital as well as in employment. The change in employment is determined by inflows in and outflows out of unemployment. The inflows into unemployment are characterized by the separation of existing job-matches at any point in time and are described by the exogenously given separation rate $\nu$ times the workers $E$. Thus, inflows characterize the number of unproductive jobs which generate layoffs.[16] On the other hand, outflows are represented by the flow of newly formed job-matches and, therefore, by the matching-function $m(U, V; \hat{\lambda})$. Firms create and offer new productive jobs and they have to fill these vacancies by searching for suitable workers. At the aggregate level, the filling of vacancies depends on the number of unemployed, the number of offered vacancies, the search intensities of firms and unemployed and the rate of technical progress; all determinants are expressed in the matching-function. Taking outflows and inflows together, the dynamics of employment result as the difference between both and can be expressed as

$$\dot{E} = m(U, V; \hat{\lambda}) - \nu E. \tag{11}$$

Each vacancy induces search costs of $c_v$ with $c_v := c_{v0} e^{\hat{\lambda} t}$. Since the newest jobs contain the latest technology, it is costly for the firm to find unemployed workers being able to handle most recent technologies. Therefore, search costs grow with the rate of technical progress.

Taking these aspects into consideration, the representative firm faces the following intertemporal optimization problem with the current flow of profits as output minus factor payments minus search expenditures. The factor payments consists of cost for rented capital $rK$, labor cost $wE$ and import cost $p_z Z$. Denoting $r$ as the discount factor, the firm's maximization can be written as

$$\max_{I,V,Z} \int_0^\infty \{F(K, \lambda E, Z) - rK - wE - p_z Z - c_v V\} e^{-rt} dt$$

$$\begin{aligned} \text{s.t.} \qquad & \dot{E} = m(U, V; \hat{\lambda}) - \nu E \\ & \dot{K} = I \\ & K(0), E(0) \quad \text{given.} \end{aligned} \tag{12}$$

In order to solve the optimization problem, a present-value Hamiltonian function $\mathcal{H}(K, E, V, I, \mu_1, \mu_2)$ with two state variables, $E$ respectively $K$, the control variables $I$ and $V$ as well as costate variables $\mu_i$ $[i = 1, 2]$ is set up. Denoting $F_j$ as the partial derivative of $F(\cdot)$ with respect to $j = K, E, Z$, the Hamiltonian conditions are

$$\frac{\partial \mathcal{H}}{\partial V} = 0 \iff -e^{-rt} c_v + \mu_1 m_V = 0 \tag{13}$$

$$-\dot{\mu}_1 = \frac{\partial \mathcal{H}}{\partial E} \iff -\dot{\mu}_1 = e^{-rt}[F_E - w] - \mu_1 \nu \tag{14}$$

$$\dot{E} = \frac{\partial \mathcal{H}}{\partial \mu_1} \iff \dot{E} = m(U, V) - \nu E \tag{15}$$

$$\frac{\partial \mathcal{H}}{\partial I} = 0 \iff \mu_2 = 0 \tag{16}$$

$$-\dot{\mu}_2 = \frac{\partial \mathcal{H}}{\partial K} \iff -\dot{\mu}_2 = e^{-rt}[F_K - r] \tag{17}$$

$$\dot{K} = \frac{\partial \mathcal{H}}{\partial \mu_2} \iff \dot{K} = I \tag{18}$$

$$\frac{\partial \mathcal{H}}{\partial Z} = 0 \iff -e^{-rt}[F_Z - p_z] = 0 \tag{19}$$

[16] For an exogenous separation rate see also Pissarides (1990) as well as Postel-Vinay (1998) and for an endogenous rate see Mortensen/Pissarides (1994, 1998).

with the transversality conditions

$$\lim_{t\to\infty} \frac{\partial \mathcal{H}}{\partial E} E = \lim_{t\to\infty} \frac{\partial \mathcal{H}}{\partial K} K = 0.$$

The first order conditions for capital, labor and imports are given by

$$F_K(k) = r \tag{20}$$

$$F_E(k) = w + \frac{\hat{\lambda}}{1-\beta} c_v \left[ r - \hat{\lambda} + \beta \left( \hat{U} - \hat{V} \right) + \nu \right] \theta^{\beta} \tag{21}$$

$$F_Z(k) = p_z \tag{22}$$

with $F_j(k)$ $[j = K, E, Z]$ as the marginal products of their respective factors and the right hand sides are marginal costs. Note that the representative firm takes the wage as given so that the firm cannot influence the wage rate in this stage. Since there are many firms, they compete with each other and, therefore, cannot determine the wage. This is done later in the wage bargaining process.[17] Furthermore, the import price as well as the interest rate are fixed due to the small country assumption.

Notice that the workers get only a fraction of the marginal product of labor which is $w = \omega F_E(k)$ and the firm receives $(1-\omega)F_E(k)$. The latter one uses it to finance its search costs and, therefore, the restriction $(1-\omega)F_E(k) = c_v V$ holds.

*International Capital Mobility*

After describing the intertemporal optimization problem of the representative firm, international financial assets have to be introduced to generate an intertemporal allocation of goods. Taub (1999, p. 492) characterizes international trade in goods and capital as follows: "...countries should trade intertemporally; the exchange will take the visible form of assets exchanged for goods, and subsequent repayment of goods. By this logic countries should be borrowing and lending in the international market." Domestic capital $K$ and net assets $B$ are assumed to be the only forms of national wealth, $A$

$$A - K = B. \tag{23}$$

$B$ is positive, that is the domestic stock of net assets is positive, if the domestic country is net creditor, $B > 0$, and vice versa. Because of international purchases or sales of net assets, net interest payments have to be regarded. National income, $Y$, consists of domestic income, $Y^d$, and net interest income, $rB$:

$$Y := rK + wE + rB, \tag{24}$$

where $rK$ and $wE$ is the domestic income, $Y^d$, with $w$ as wage rate and $r$ as world interest rate. National income would decrease with an increasing stock of net foreign debts (i.e. $B < 0$), because of rising interest payments.

As a next step the balance of payments, consisting of current account and capital account, has to be regarded. With international capital mobility, the trade account restriction does not longer hold. Beside exports, $Ex$, with the price of exports normalized to one, and imports, $p_z Z$, the current account includes in its simplest form net interest income, $rB$:

$$CA = Ex - p_z Z + rB. \tag{25}$$

Also, the current account restriction does not hold any longer.

With intertemporal exchange possibilities, the exchange restriction that has to hold is given by the balance of payments

$$Ex - p_z Z + rB - \dot{B} = 0, \tag{26}$$

[17] For the wage determination see equation (7).

where the balance of the capital account, $\dot{B}$, has to be equal to the balance of the current account, $CA$. If the country is a net creditor, it exports goods and accumulates additional net assets.

*Equilibrium of the Wealth Market*

In the small open economy with capital mobility, the savings-investment relation, being valid in a closed economy, does not hold any longer. In an economy with integrated capital markets, savings can be higher or lower than investments and the difference is characterized by the current account balance. Therefore, the equilibrium of the goods market in a small open economy is given by:

$$S = I + Ex - p_z Z + rB. \tag{27}$$

This equation substitutes the equilibrium condition for a closed economy. Substituting (26) and (27) into another, the equilibrium condition for the wealth market can be rewritten as:

$$S = I + \dot{B}. \tag{28}$$

Note that the change in net wealth is identical to domestic savings $\dot{A} \equiv S$ where domestic savings are a constant fraction of income

$$S = sY. \tag{29}$$

In the case that the country runs a current account deficit investments are build up by the abandonment of domestic as well as foreign consumption possibilities.

The domestic budget restriction consists of national income, $Y$, import cost, $p_z Z$, search cost, $c_v V$, and interest income or payment, $rB$:

$$X = Y + p_z Z - rB + c_v V, \tag{30}$$

where $X$ is the output. In the budget restriction the interest income have to be subtracted, since it is already included in national income.

# 3 Steady-State Solution

Analyzing the overall steady-state solution for the economy, the long-run equilibrium for the labor market and the steady-state for the wealth market are derived separately in a first step. They are characterized by two functions: the *efficient factor allocation function* and the *balanced accumulation function*. The first function will be the $\dot{\theta} = 0-$curve describing the stationary labor market and the second function will be the $\dot{a} = 0-$curve that represents the stationary wealth market. In a second step, after determining the stationary equilibrium in both markets separately, the stationary overall steady-state in the small open economy will be derived.

*Steady-State of the Labor Market*

The steady-state of the labor market is deduced by using the flow condition of the labor market. This condition requires that the inflows are equal to the outflows and, therefore, the change in employment is zero:

$$\dot{E} = 0 \;\Leftrightarrow\; V^{1-\beta} U^{\beta} \hat{\lambda}^{-1} = \nu E. \tag{31}$$

Furthermore, due to neglecting on-the-job-search, the flow of newly created vacancies is identical to the employment flow, i.e. $\dot{V} = \dot{E} = 0$, and because of a constant labor force, the employment and unemployment levels are constant in the long-run equilibrium, i.e. $\dot{E} = -\dot{U} = 0$. These conditions imply that steady-state labor market tightness is also constant, i.e. $\dot{\theta} = 0$, and that the steady-state growth rates of unemployment and vacancies are zero, i.e. $\hat{V} = \hat{U} = 0$.

Using these conditions, the *efficient factor allocation function* for the stationary labor market can be derived:[18]

$$\Psi(a) := \frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\lambda_o}{\widehat{\lambda} c_{v0}\left[r+\nu-\widehat{\lambda}\right]}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} = \theta^{\beta} \tag{32}$$

It shows all combinations of wealth in efficiency units and labor market tightness that reflect the long-run equilibrium of the labor market in a small open economy, i.e. this function is the $\dot{\theta} = 0-$curve. The steady-state of the labor-market is influenced by several exogenous variables and it will change when the exogenous environment changes. In the $(\theta^{\beta}, a)$ plane it is linear in $a$ with $\Psi'(a) = 0$, $\forall a$.[19]

Furthermore, in the long-run labor market equilibrium the steady-state employment rate is given by[20]

$$e(\theta) := \frac{E}{\bar{L}} = \frac{p(\theta)}{\nu + p(\theta)}, \quad e_{\theta} > 0. \tag{33}$$

Therefore, the employment probability depends positively on labor market tightness $\theta$ and on the matching-probability $p(\theta)$ and negatively on the separation rate $\nu$. The higher the separation rate, the lower the steady-state employment rate. Furthermore, the steady-state unemployment rate is determined as well as

$$1 = e(\theta) + u(\theta),$$

where the steady-state unemployment rate $u(\theta)$ is defined as $u(\theta) := U/\bar{L}$. The higher the separation rate, the lower the steady-state employment rate and the higher the steady-state unemployment rate. Thus, the steady-state for the labor market is described by an efficient factor allocation function that defines all equilibrium combinations of labor market tightness and wealth in efficiency units.

*Steady-State of the Wealth Market*

The long-run steady-state of the wealth market is characterized by constant wealth in efficiency units, $a := \frac{A}{\lambda E} = \frac{K+B}{\lambda E}$, i.e.

$$\dot{a} = \frac{\dot{K} + \dot{B}}{\lambda E} - (\hat{\lambda} + \hat{E})\frac{A}{\lambda E} = 0. \tag{34}$$

In the steady state, the wealth accumulation process in labor efficiency units is constant; that is, domestic accumulation wishes $(\hat{\lambda} + \hat{E})a$ are build up by the same accumulation structure of both assets in efficiency units $(\dot{k} + \dot{b})$. Hence, the domestic demand for investments is equal to the constant accumulation structure. The steady-state of the wealth market can be described by a *balanced accumulation function*:[21]

$$\Phi(a) := \frac{\lambda_0}{c_{v0}\hat{\lambda}\nu}\left\{(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} + \left(sr - \hat{\lambda}\right)a\right\} = \theta^{\beta} \tag{35}$$

This function shows all combinations of labor market tightness and wealth in efficiency units characterizing the steady-state in the wealth market. That is, this function represents the $\dot{a} = 0-$curve. Satisfying $sr - \hat{\lambda} < 0$, the balanced accumulation function is linear in $a$ with $\Phi'(a) < 0$, $\forall a$ in the $(\theta^{\beta}, a)$ plane.[22]

[18] For the detailed derivation of the efficient factor allocation function see appendix, proposition 6.
[19] See appendix, proposition 7.
[20] See appendix, proposition 8.
[21] For a detailed derivation of the balanced accumulation function see appendix, proposition 9.
[22] See appendix, proposition 10.

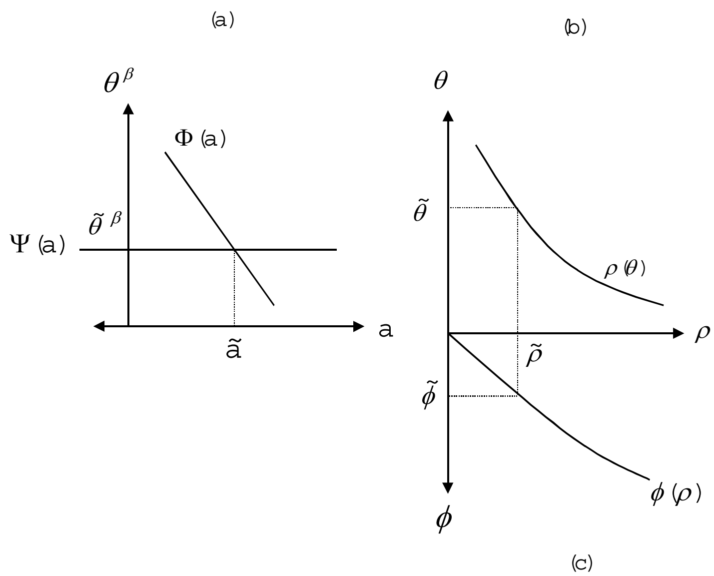


Figure 1: *Steady-State of a Net Creditor Economy.*

## 3.1 Overall Steady-State of the Economy

After deriving the equilibrium conditions for the labor market respectively for the goods market separately, both together determine the overall steady-state, i.e. the efficient factor allocation function and the balanced accumulation function simultaneously define the steady-state values $(\tilde{\theta}^{\beta}, \widetilde{a})$. The steady-state exists and is unique.[23]

**Proposition 1** *If the net wealth condition*

$$\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{[\nu-\hat{\lambda}+r]}+\alpha+\gamma\geq 1 \tag{36}$$

*is satisfied in the stationary equilibrium, the domestic country will be a net creditor with positive net wealth in efficiency units, i.e.* $\widetilde{a}\geq 0$.

**Proof.** See appendix, proposition 12. ■

If the net wealth condition holds, the domestic country will be a net creditor and is accumulating in the long-run steady-state constantly net wealth in efficiency units with $\widetilde{a}\geqslant 0$ (see Figure 1a). Therefore, the domestic country owns a stationary net wealth proportion on the whole net wealth of the rest of the world. Violating this condition, i.e., if

$$\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{[\nu-\hat{\lambda}+r]}+\alpha+\gamma< 1 \tag{37}$$

is valid, the rest of the world is a net creditor, the domestic country is net debtor and net wealth in efficiency units will be negative ($\widetilde{a}<0$). Therefore, the balanced accumulation function as well as the overall steady-state equilibrium $(\tilde{\theta}^{\beta}, \widetilde{a})$ will be in left quadrant in Figure 1a.

Once the steady-state search equilibrium $(\tilde{\theta}^{\beta}, \widetilde{a})$ is determined, the steady-state values for the matching probability $\tilde{p}$, the steady-state employment respectively unemployment rate $\tilde{e}$ respectively $\tilde{u}$ can be derived. The

[23] See appendix, proposition 11.

steady-state employment and unemployment levels are fixed as well: $\tilde{E} = e(\tilde{\theta})\bar{L}$ and $\tilde{U} = u(\tilde{\theta})\bar{L}$. Furthermore, steady-state labor market tightness determines the equilibrium unemployment duration $\tilde{\rho}$ and the steady-state fraction of the long-term unemployed $\tilde{\phi}$ (see Figure 1b,c).

*Behavior of the Trade Account and Current Account in the Steady-State*

After deriving the overall steady-state of the economy, the long run equilibrium positions of the current account and the trade account can also be ascertained. Looking at the idea of long run well-balanced trade, this model does not support off-setting steady-state trade, i.e. even in the long run, countries do not have to have well-balanced trade. This unbalanced long run trade can be characterized by permanent current account deficits (surplus') sustained in the steady state. To derive this, the steady-state current account deficit (surplus) in efficiency units is defined as $\widetilde{ca}$ and is determined by the growth rate of technical progress and the steady state value of net wealth[24]

$$\widetilde{ca} = \hat{\lambda}\widetilde{a}. \tag{38}$$

This equation shows that even in steady-state the current account is not well-balanced, i.e. $\widetilde{ca} \neq 0$. Depending on the net creditors' or net debtors' position, a permanent current account surplus or deficit is sustained in the steady-state. For example satisfying (37), the country is a net debtor with a constant current account deficit in the long run, i.e. $\widetilde{ca} = \hat{\lambda}\widetilde{a} < 0$. In absolute terms, however, the steady state current account deficit increases. Furthermore, as steady-state accumulation wishes of the domestic country increase, i.e. $\hat{\lambda}\widetilde{a}$ rises, the steady-state current account deficit in efficiency units will increase as well.

Interesting is also the question, whether, for example, net capital exports to accumulate wealth are financed by a trade account deficit or surplus or by international interest income.[25] The steady-state trade balance in efficiency units, $\widetilde{tb}$, is defined as the steady-state current account balance in efficiency units, $\widetilde{ca}$, minus international interest income in efficiency units, $r\widetilde{a}$, i.e.

$$\widetilde{tb} = \widetilde{ca} - r\widetilde{a} = \left(\hat{\lambda} - r\right)\widetilde{a}.$$

As long as the trade balance is counterbalanced ($\widetilde{tb} = 0$), net wealth accumulation is exclusively financed by steady-state interest income. Furthermore, as the current account shows a surplus, the steady-state trade account does not necessarily have to be positive, since it can be well-balanced by interest income, i.e. $\widetilde{tb} = \underbrace{\widetilde{ca}}_{>0} - r\underbrace{\widetilde{a}}_{>0} \gtreqless 0$. However, for a net creditor country, a trade deficit can exists as long as the domestic growth rate of technical progress is less than the world interest rate. Putting it differently: If the net creditors' accumulation wishes are smaller than international interest income, the steady-state trade account has a long run deficit and imports are higher than exports. Therefore, in the steady-state, the domestic country receives international interest income to finance its net good imports. Hence, with the introduction of capital mobility, a net creditor country can use permanently resources from abroad and obtains simultaneously continuous international interest income (and vice versa for a net debtor position).

## 3.2 Stability of the Steady-State

*Global Stability*

[24] See appendix, proposition 13.

[25] See equation (26).

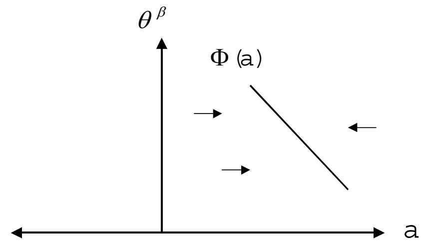


Figure 2: *Transitional Dynamics for Net Wealth in Efficiency Units.*

The transitional behavior of net wealth in efficiency units is characterized by the dynamic wealth accumulation function[26]

$$\begin{aligned} \dot{a} &= f(a,\theta) \qquad (39) \\ &= s\left\{(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}-\frac{c_{v0}\hat{\lambda}(\hat{E}+\nu)\theta^{\beta}}{\lambda_0}\right\}+\left(sr-\hat{\lambda}\right)a, \end{aligned}$$

and the global properties can be analyzed using the following expressions

$$\dot{a} < 0 \Leftrightarrow \theta^{\beta} > \frac{\lambda_0}{c_{v0}(\hat{E}+\nu)\nu}\left\{(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}+\left(sr-\hat{\lambda}\right)a\right\}$$

and vice versa. If a realized value for the labor market tightness for given net wealth in efficiency units is higher than the steady-state value for labor market tightness, the firm is offering too many vacancies for a given level of unemployment. The supply of vacancies has to be reduced to be consistent with the long-run growth equilibrium. Cutting back vacancies implies that capital accumulation and, therefore, the net wealth position is shrinking. Hence, above the $\dot{a}=0-$curve, too many vacancies are supplied and capital accumulation as well as net wealth accumulation in efficiency units has to be reduced to reach the long-run steady-state (see Figure 2). That is, if a value of labor market tightness is realized which is higher than the equilibrium value of labor market tightness, net wealth in efficiency units should decrease to reach the steady-state, i.e. $\dot{a}<0$.

Furthermore, the transitional dynamics of the labor market tightness are given by the dynamic factor allocation function[27]

$$\begin{aligned} \dot{\theta} &= h(a,\theta) \qquad (40) \\ &= \frac{\left[r+\nu-\hat{\lambda}\right]\theta}{\beta}-\frac{(1-\alpha-\gamma)(1-\omega)(1-\beta)\lambda_0}{c_{v0}\hat{\lambda}\beta\theta^{\beta-1}}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}. \end{aligned}$$

Analyzing the global dynamic properties of the dynamic factor allocation function, labor market tightness for given net wealth in efficiency units increases, if values of the labor market tightness are realized being above the $\dot{\theta}=0$-curve:

$$\theta^{\beta} > \frac{(1-\alpha-\gamma)(1-\omega)(1-\beta)\lambda_0}{c_{v0}\hat{\lambda}\left[r+\nu-\hat{\lambda}\right]}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}},$$

which implies $\dot{\theta}>0$ and vice versa (see Figure 3). If a value for the labor market tightness is realized being higher than the equilibrium value for given $\beta$ and given net wealth in efficiency units, labor market tightness

[26] See appendix, proposition 15.
[27] See appendix, proposition 14.

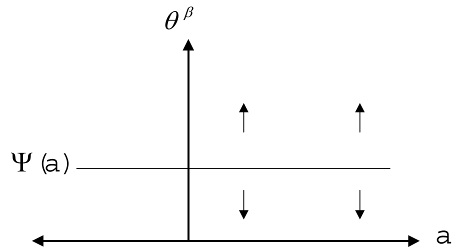


Figure 3: *Transitional Dynamics of Labor Market Tightness.*

will increase and will never reach the stable long-run equilibrium. Only if values for labor market tightness are realized that are on the factor allocation function, labor market tightness will not drift away and approaches the steady-state. That is, labor market tightness is not a predetermined variable; instead it is the jump variable of the model.

Putting the global properties of labor market tightness and net wealth in efficiency units together, the global dynamics in Figure 4 are implied.

*Local Stability*

After characterizing global stability, the local properties can be examined by linearizing the dynamic system in the area of the steady state, i.e.

$$\begin{bmatrix} \dot{\theta} \\ \dot{a} \end{bmatrix} = \underbrace{\begin{bmatrix} h_a & h_\theta \\ f_a & f_\theta \end{bmatrix}}_{:=H} \begin{bmatrix} \theta - \widetilde{\theta} \\ a - \widetilde{a} \end{bmatrix}$$

with

$$\begin{aligned} h_a &= 0 \\ h_\theta &= \frac{r+\nu-\hat{\lambda}}{\beta} - \frac{(1-\beta)^2(1-\alpha-\gamma)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\beta q(\theta)} \left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} \\ &= \; ? \\ f_a &= sr - \hat{\lambda} - \hat{E} < 0 \\ f_\theta &= -\frac{c_{v0}\hat{\lambda}s\left(\hat{E}+\nu\right)\beta\theta^{\beta-1}}{\lambda_0} < 0, \end{aligned}$$

where $\widetilde{\theta}$ and $\widetilde{a}$ represent the steady-state values of the model.

**Proposition 2** *Suppose*

$$\begin{aligned} h_\theta &= \frac{r+\nu-\hat{\lambda}}{\beta} - \frac{(1-\beta)^2(1-\alpha-\gamma)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\beta q(\theta)} \left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} \\ &< 0, \end{aligned}$$

*the unique steady-state is a saddle point.*

**Proof.** See appendix, proposition 16.[28] ■

[28] See also Leonard/Van Long (1992, p. 100-1) and Pissardies (1990, p. 45).

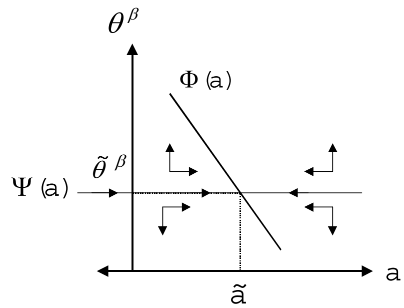


Figure 4: *Stability in an open Growth-Matching-Model.*

The saddle point equilibrium is characterized by the saddle path leading in the locally stable steady state. In Figure 4 the saddle path is represented by the efficient factor allocation function, $\Psi(a)$. If an economy is on this function, labor market tightness is constant and labor market structures are not changing anymore. The steady state can only be reached by being on the stable saddle path and the economy has to jump on it. If the economy is on the stable saddle path, the structures of the labor market are constant, however, the domestic wealth situation as a net creditor respectively net debtor can change further. Due to additional capital imports, the domestic country can move from a net creditor to a net debtor. The changes in the accumulation process of domestic net debt titles induces also changes in the current account. As long as the asset structures of net debt titles are changing, i.e. as long as the economy has not reached the long-run steady-state growth equilibrium, the economy will experience current account changes. With increasing net asset accumulation of the rest of the world, the domestic country has to carry higher interest payments and, therefore, it is confronted with a higher current account deficit.

# 4 Economics of the Steady-State

After deriving the long-run steady-state as well as the stability of the model, economic implications especially the effects of rising technical progress on the labor market are analyzed in this section. In the first situation of fulfilling a parametric growth condition which requires that the growth rate of technical progress is larger than the average of interest rate and separation rate, the implications of rising productivity growth on the equilibrium levels of employment, unemployment and the duration of unemployment as well as on the steady-state fraction of long-term unemployment are analyzed. After that, in the second case, the effects of a rise of technical progress on these variables will be described when violating the growth condition.

## 4.1 The Growth Condition

For analyzing the effect of an increase in the growth rate of technical progress $\hat{\lambda}$ on steady-state employment, unemployment, the equilibrium duration of unemployment and the steady-state fraction of long-term unemployment, its influence on the efficient factor allocation function, on the balanced accumulation function as well as on the overall steady-state will be derived.

**Proposition 3** *If the growth rate is larger than the average of world interest rate and separation rate, that is, if the growth condition $\hat{\lambda} > \frac{r+\nu}{2}$ is valid, increasing technical progress induces a positive capitalization effect such that steady state employment increases. However, because of negative acting stigmatization and human capital* *depreciation effects, a trade-off between technical progress and the steady-state duration of unemployment as well as the steady-state fraction of long-term unemployment arises. Therefore, it is not clear whether equilibrium long-term unemployment will raise or shrink.*

This can be shown by analyzing the implication of an increase in the growth rate of technical progress for the efficient factor allocation function and the balanced accumulation function. As long as the growth condition $\hat{\lambda} > \frac{r+\nu}{2}$ holds, an increase in the rate of technical progress causes an increase in labor market tightness. That is, the labor market becomes less tight and for given net wealth labor market tightness increases

$$\begin{aligned}\frac{\partial \Psi(a)}{\partial \hat{\lambda}} &= -\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\lambda_0 \left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}}{c_{v0}\hat{\lambda}^2 \left[\nu - \hat{\lambda} + r\right]^2} \underbrace{\left[\nu - 2\hat{\lambda} + r\right]}_{<0} \\ &> 0.\end{aligned}$$

The factor allocation function shifts upwards. Furthermore, analyzing the effect of increasing productivity growth on the balanced accumulation function, a negative relationship is derived:

$$\frac{\partial \Phi(a)}{\partial \hat{\lambda}} = -\frac{\lambda_0}{c_{v0}\nu\hat{\lambda}^2}\left[(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} + sra\right] < 0$$

The increase in the growth rate of technical progress affects the balanced accumulation negatively and for given net wealth the labor market becomes tighter, i.e. labor market tightness decreases. This is reflected by the downward shift of the balanced accumulation function.

To examine the total effect of rising technical progress on the equilibrium positions of the economy, the above implications–both derived separately–have to be analyzed together. Overall, rising productivity growth will induce a higher level of offered vacancies for given unemployment, and jobless workers will face a higher matching rate. Therefore, steady-state unemployment will decrease which leads for given vacancies to an increase in steady-state labor market tightness defined as the ratio of vacancies to unemployment. However, the effects on the steady-state duration of unemployment as well as the steady-state fraction of long-term unemployment are ambiguous.

These overall effects can be shown clearly by regarding three different effects. First, the increase in steady-state labor market tightness can be explained by the so-called *capitalization effect* to indicate that higher technical progress is favorable for employment, due to the net creation of jobs. According to Aghion, Howitt (1994) technical progress is not equally distributed among jobs and firms, implying that technical progress generates and destroys jobs simultaneously. They require that, at low growth rates, increasing technical progress causes via the net destruction of jobs a creative destruction effect that leads to increasing unemployment; while, at high growth rates, increasing technical progress generates a capitalization effect that dominates the negative creative destruction effect to result in rising employment through the net creation of jobs.

In this model independent of the level of the growth rate, the capitalization effect occurs as well, but it depends – opposite to Aghion, Howitt (1994) – on the above growth condition. In the steady-state, boosting technical progress leads to the net creation of new jobs to induce a higher equilibrium level of vacancies and, therefore, a higher level of labor market tightness (see Figure 5a). That is, for given vacancies the increase in steady-state labor market tightness is equivalent to reduced steady-state unemployment and equilibrium employment will raise.

Furthermore, regarding the net creditor or net debtor position of the country, higher productivity growth implies in the case of fulfilling the growth condition a lower steady-state net wealth position due to the negative shape of the balanced accumulation function (from $\widetilde{a}_0$ to $\widetilde{a}_1$, see Figure 5a).

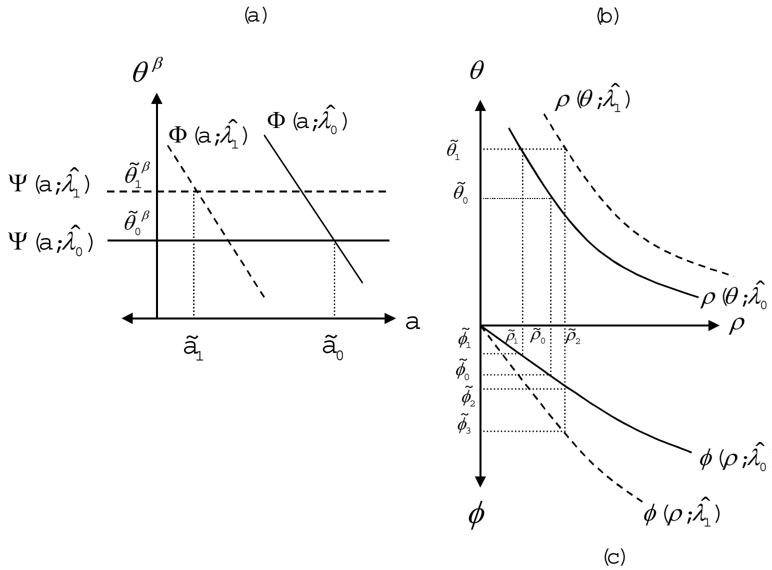


Figure 5: *Steady-State Implications for a Net Creditor by Increasing Technical Progress and Fulfilling the Growth Condition.*

To analyze the implications of increasing technical progress on the steady-state duration of unemployment and on the steady-state fraction of long-term unemployment, negative acting secondary effects – the stigmatization effect and the human capital depreciation effect – are induced. The increase in the steady-state unemployment duration resulting from an increase in the productivity rate can be explained by the so-called *stigmatization effect.* Because the unemployment pool consists of short-term and long-term unemployed and because long-term unemployed are stigmatized by long jobless durations that indicate their long time being out of work and less working experience, they are not seen as potential candidates for unfilled jobs and are not demanded by firms.[29] Firms want to hire only short-term jobless workers with short unemployment durations. Therefore, as productivity growth goes up, the duration of unemployment increases for all duration levels implying a higher steady-state unemployment duration (see Figure 5b).

Third, the *human capital depreciation effect* will be explained. In growing economies human capital depreciation is a decisive factor for the existence and the increase of long-term unemployment and can be described as follows. The higher the growth rate of technological progress, the more qualification-intensive and specific the job-requirements of firms are.[30] If the qualification level of the long-term unemployed does not grow with the same rate as technical progress, their human capital depreciates as technical progress increases and it depreciates faster, the longer the unemployment duration takes. The long-term unemployed cannot handle the latest technologies, are not attractive for firms and are not demanded. Therefore, an increase in technical progress leads to higher levels of long-term unemployment for all unemployment durations and, hence, to a higher steady-state long-term unemployment fraction (see Figure 5c).

To summarize these three different effects, an increase in technical progress induces a capitalization effect such that labor market tightness goes up (from $\widetilde{\theta}_0^{\beta}$ to $\widetilde{\theta}_1^{\beta}$, see Figure 5a). The positive capitalization effect leads to an increase in steady-state employment respectively to a decrease in steady-state unemployment (i.e. for given vacancies to an increase in labor market tightness from $\widetilde{\theta}_0$ to $\widetilde{\theta}_1$, see Figure 5b) and to reductions in the

[29] See also Layard (1986, p. 53), who argues "...that long-term unemployment is a complete waste."
[30] For mismatch due to qualificational reasons, see also Juhn, Murphy, Topel (1991) and Blanchard, Katz (1997).

unemployment duration (from $\widetilde{\rho}_0$ to $\widetilde{\rho}_1$) as well as in the fraction of long-term unemployed (from $\widetilde{\phi}_0$ to $\widetilde{\phi}_1$). Despite this positive capitalization effect that induces a reduction in unemployment, long-term unemployment as well as in the duration of unemployment, two negative acting effects – the stigmatization and the human capital depreciation effect – are generated additionally. The stigmatization effect implies a higher unemployment duration (from $\widetilde{\rho}_1$ to $\widetilde{\rho}_2$, see Figure 5b) and this induces an increase in the fraction of long-term unemployed (from $\widetilde{\phi}_1$ to $\widetilde{\phi}_2$, see Figure 5c). Furthermore, as technical progress increases, the fraction of long-term unemployed rises additionally (from $\widetilde{\phi}_2$ to $\widetilde{\phi}_3$) because of the human capital depreciation effect. Therefore, it is not clear how the duration of unemployment and the fraction of long-term unemployment are affected by the increase in productivity growth; ambiguous effects for the duration of unemployment and the fraction of long-term unemployment can result. If the capitalization effect outweighs the stigmatization and human capital depreciation effect, the duration of unemployment as well as the fraction of long-term unemployment will decrease. However, if both latter effects outweigh the positive capitalization effect, the duration of unemployment and the fraction of long-term unemployment will increase, although steady-state unemployment decreases.

## 4.2 Violating the Growth Condition

**Proposition 4** *If, on the other hand, the average of the interest rate and separation rate are larger than the growth rate, i.e. if the condition $\hat{\lambda} < \frac{r+\nu}{2}$ holds, an increase in the rate of technical progress induces a negative creative destruction effect such that steady-state unemployment will increase which is reinforced by the stigmatization effect as well as the qualification-mismatch effect. Therefore, equilibrium unemployment and the steady-state fraction of long-term unemployment will unambiguously raise.*

The increase in technical progress can be shown by analyzing, first, its effect on the factor allocation function and on the balanced accumulation function and, second, its overall effect on the steady-state positions. If the growth condition does not hold any longer, i.e. the growth rate is smaller than the average of fixed interest rate and separation rate, the labor market becomes tighter and labor market tightness decreases

$$\begin{aligned}\frac{\partial \Psi(a)}{\partial \hat{\lambda}} &= -\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\lambda_0 \left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}}{c_{v0}\hat{\lambda}^2 \left[\nu-\hat{\lambda}+r\right]^2} \underbrace{\left[\nu - 2\hat{\lambda} + r\right]}_{>0} \\ &< 0.\end{aligned}$$

With rising technical progress, labor market tightness decreases such that firms are offering less vacancies and, as a result, the steady-state level of vacancies will shrink for a given level of unemployment. The factor allocation function shifts downward. For an average unemployed worker it becomes more difficult to leave unemployment. The effect on the balanced accumulation function, however, does not change and the negative relationship between increasing technical progress and steady-state labor market tightness is still valid

$$\frac{\partial \Phi(a)}{\partial \hat{\lambda}} = -\frac{\lambda_0}{c_{v0}\nu\hat{\lambda}^2}\left[(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} + sra\right] < 0,$$

i.e., labor market tightness will decrease for all levels of net wealth with rising technical progress.

To examine raising technical progress when violating the growth condition, the overall effects on the equilibrium positions have to be analyzed. When violating the growth condition, the same stigmatization and human capital depreciation effects are implied as before. However, the positive capitalization effect leading to raising employment is reversed into a negative Schumpeterian *creative destruction effect*. As in Aghion/Howitt (1994),

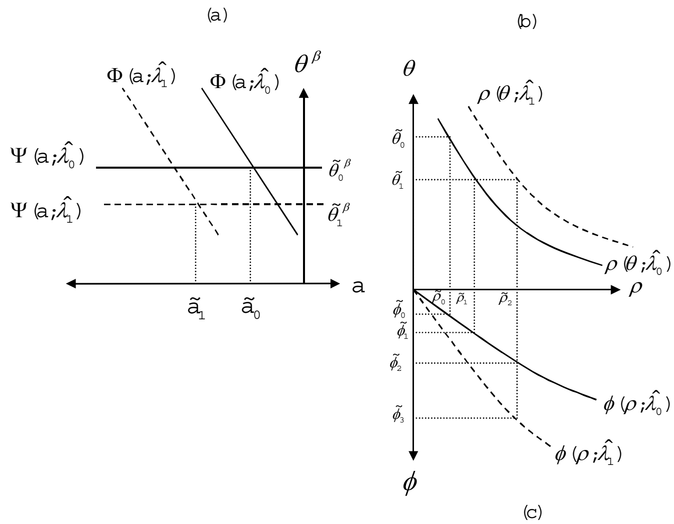


Figure 6: *Steady-State Implications for a Net Debtor by Increasing Technical Progress and Violating the Growth Condition.*

the creative destruction effect indicates that more jobs are destroyed than new jobs are created when technological progress rises. This causes the net destruction of jobs and, in equilibrium, unemployment increases which implies a reduction in labor market tightness defined as the ratio of vacancies to unemployment. Therefore, higher productivity growth implies the net destruction of jobs as well as higher steady-state unemployment such that labor market tightness reduces (from $\widetilde{\theta}_0^\beta$ to $\widetilde{\theta}_1^\beta$, see Figure 6a).

Regarding the net creditor or net debtor position of the country, higher productivity growth could imply a higher steady-state net wealth position, but also a lower steady-state net wealth position. Hence, net wealth can rise or shrink with increasing technical progress.

Analyzing the implications for the steady-state duration of unemployment and for the steady-state fraction of long-term unemployment, the stigmatization as well as the human capital depreciation effects are induced. Due to the negative creative destruction effect, unemployment increases, i.e. for given vacancies labor market tightness decreases (from $\widetilde{\theta}_0$ to $\widetilde{\theta}_1$, see Figure 6b), which implies that it becomes more difficult for an average unemployed to leave unemployment and steady-state duration of unemployment rises (from $\widetilde{\rho}_0$ to $\widetilde{\rho}_1$). The long-term unemployed are stigmatized by long jobless durations and, as technical progress increase, they are even less able to use the latest technologies. Therefore, due to the stigmatization effect, the duration increases additionally (from $\widetilde{\rho}_1$ to $\widetilde{\rho}_2$) leading to a higher fraction of long-term unemployment (from $\widetilde{\phi}_1$ to $\widetilde{\phi}_2$, see Figure 6c). Furthermore, since the long-term unemployed are not additionally trained and qualified as technical progress rises, their human capital depreciates even more. They are not demanded by the firms at all and, because of the human capital depreciation effect, the steady-state fraction of long-term unemployment will increase further (from $\widetilde{\phi}_2$ to $\widetilde{\phi}_3$).

To summarize, if the growth condition is violated, increasing technical progress causes rising unemployment and a net destruction of jobs such that the creative destruction effect is reinforced by the stigmatization and the human capital depreciation effect inducing higher steady-state unemployment duration as well as higher equilibrium long-term unemployment.

## 5 Conclusion

In this contribution, I analyzed the effect of high rates of technical progress on long-term unemployed workers in an open economy, where growth is not restricted by any trade boundaries. Depending on a growth condition, I showed that an increase in growth rate implies a favorable capitalization effect inducing the net creation of jobs and shrinking average unemployment and, on the other hand, an unfavorable creative destruction effect causing the net destruction of existing jobs and increasing average unemployment. My conclusions confirm the ambiguous effects of growth on average unemployment found in the literature, but extend it by deriving a negative relation between the growth rate and the fraction of long-term unemployed workers. One assumption underlying the adverse relation is the easy access and fast distribution of new technologies across countries, which is reflected by the open economy set-up. Through the fast spread of technologies, a high rate of growth induces stigmatization and human capital depreciation effects for long-term unemployed. Stronger productivity growth implies higher durations particularly for long-term unemployed workers. Due to high technological progress, firms demand for their newly created vacancies endowed with the most recent technologies workers that are able to handle these latest technologies. Since the human capital of long-term unemployed does not fit those requirements, they are firstly not demanded by firms, remain therefore longer in the unemployment pool than short-time unemployed and are stigmatized by high durations of unemployment. Since they are not demanded by firms, their human capital depreciates—and it depreciates even faster the higher the rates of technical progress. That is, growth creates new vacancies, but this employment generating capitalization effect is outweighed by the stigmatization and human capital depreciation effects, which both act negatively on long-term unemployed workers. The paper explains therefore the rise in long-term unemployment, not by the increase or high levels of benefit payments on which the literature usually focuses, but through the implications high growth rates have on long-term unemployed workers. Since the mid 1980s, this strong growth has occurred in *all* industrialized countries—in Europe as well in the U.S.—which is partly accountable for high long-term unemployment in those countries.[31]

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[31] See Figure 7 in the Appendix.

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# Appendix

**Lemma 5** *Using the matching function (2), the Hamiltonian conditions (13), (14),* $c_v = c_{v0}e^{\hat{\lambda}t}$ *and* $\lambda = \lambda_0 e^{\hat{\lambda}t}$*, the optimal condition for labor (21) is implied.*

**Proof.** Differentiate (2) w.r.t. $V$ and (13) w.r.t. time, use $c_v := c_{v0}e^{\hat{\lambda}t}$ and $\lambda := \lambda_0 e^{\hat{\lambda}t}$, then

$$\begin{aligned} & e^{-rt}c_v\left(r-\hat{\lambda}\right)+\frac{1-\beta}{\hat{\lambda}}\left[\dot{\mu}_1+\mu_1\beta\left(\hat{U}-\hat{V}\right)\right]\theta^{-\beta}=0 \\ \Leftrightarrow\quad & -\dot{\mu}_1=\frac{\hat{\lambda}}{1-\beta}e^{-rt}c_v\left(r-\hat{\lambda}\right)\theta^{\beta}+\mu_1\beta\left(\hat{U}-\hat{V}\right). \end{aligned}$$

Substitute (14) for $-\dot{\mu}_1$, then

$$\frac{\hat{\lambda}}{1-\beta}e^{-rt}c_v\left(r-\hat{\lambda}\right)\theta^{\beta}+\mu_1\beta\left(\hat{U}-\hat{V}\right)=e^{-rt}\left[F_E(k)-w\right]-\mu_1\nu$$

and substitute (13) for $\mu_1$, then

$$\begin{aligned} & c_v\frac{\hat{\lambda}}{1-\beta}\theta^{\beta}\left[\beta\left(\hat{U}-\hat{V}\right)+\nu\right]=F_E(k)-w-c_v\frac{\hat{\lambda}}{1-\beta}\theta^{\beta}\left(r-\hat{\lambda}\right) \\ \Leftrightarrow\quad & F_E(k)=w+c_v\frac{\hat{\lambda}}{1-\beta}\left[r-\hat{\lambda}+\beta\left(\hat{U}-\hat{V}\right)+\nu\right]\theta^{\beta}. \end{aligned}$$

Therefore, the optimal condition for labor (21) is implied. ■

**Proposition 6** *Using labor restriction (1), wage hypothesis (7), production functions (8) and (9), optimal conditions for capital (20), for labor (21) as well as for imports (22) and the flow condition for employment (31), the efficient factor allocation function (32) follows.*

**Proof.** Differentiate (8) w.r.t. $Z$ and use $\varepsilon := 1-\alpha-\gamma$, then

$$F_Z(k)=\gamma z^{\gamma-1}k^{\alpha}. \tag{8'}$$

Substitute this into (22), then

$$\gamma z^{\gamma-1}k^{\alpha}=p_z, \tag{22'}$$

and (9) in (22'), then

$$\frac{\gamma}{p_z}x=z. \tag{22''}$$

Furthermore, substitute this back into (9), then

$$x=\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\gamma}}k^{\frac{\alpha}{1-\gamma}}. \tag{9'}$$

Differentiate (8) w.r.t. $K$ and $E$ and use (8'), then

$$F_K(k) = \alpha k^{-1}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}} \tag{8''}$$

$$F_E(k) = (1-\alpha-\gamma)\lambda\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}}. \tag{8'''}$$

Substitute (7) in (21), then

$$F_E(k) = \frac{\widehat{\lambda} c_v\left[r-\widehat{\lambda}+\nu+\beta(\widehat{U}-\widehat{V})\right]\theta^{\beta}}{(1-\beta)(1-\omega)} \tag{21'}$$

Equate (21') with (8''') and use $c_v = c_{vo}e^{\widehat{\lambda}t}$, $\lambda = \lambda_0 e^{\widehat{\lambda}t}$, then

$$\frac{\widehat{\lambda} c_v\left[r-\widehat{\lambda}+\nu+\beta(\widehat{U}-\widehat{V})\right]\theta^{\beta}}{(1-\beta)(1-\omega)} = (1-\alpha-\gamma)\lambda\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}}$$

$$\Leftrightarrow \quad \theta^{\beta} = \frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\lambda_o}{\widehat{\lambda} c_{v0}\left[r-\widehat{\lambda}+\nu+\beta(\widehat{U}-\widehat{V})\right]}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}} \tag{21''}$$

Substitute (8'') in (20), then

$$r = \alpha\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}-1}$$

$$\Leftrightarrow \quad k = \left(\frac{r}{\alpha}\right)^{\frac{1-\gamma}{\alpha+\gamma-1}}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \tag{20'}$$

Differentiate (1) w.r.t. time, then $\dot{E} = -\dot{U}$, use $\theta := V/U$, then $\dot{\theta} = \dot{V}/U - V\dot{U}/U^2$; use (1) and (31), then $\dot{E} = 0 = -\dot{U}$, $\dot{\theta} = 0$ and $\dot{V} = 0$ are implied and, therefore, $\hat{U} = \hat{V} = 0$. Substitute this and (20') in (21''), rearrange it, then

$$\theta^{\beta} = \frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\lambda_o}{\widehat{\lambda} c_{v0}\left[\nu-\widehat{\lambda}+r\right]}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} =: \Psi(a)$$

follows. ∎

**Proposition 7** *Suppose $\nu - \hat{\lambda} > 0$, then $\Psi(a) > 0$ and $\Psi(a)$ is a linear function with $\Psi'(a) = 0$.*

**Proof.** If $\nu - \hat{\lambda} > 0$, then

$$\Psi(a) := \frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\lambda_o}{\widehat{\lambda} c_{v0}\left[\nu-\widehat{\lambda}+r\right]}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} > 0 \ \ \forall a.$$

Furthermore, $\Psi^{'}(a) = 0$. ∎

**Proposition 8** *Using labor restriction (1), matching function (2), matching probability (3) and the flow condition for employment (31), the steady-state employment rate $e(\theta) = \frac{p(\theta)}{\nu+p(\theta)}$ is implied.*

**Proof.** Using (2), (31) can be written as

$$\dot{E} = 0 \Leftrightarrow M = \nu E$$

and using (1) as well as (3), then

$$\begin{aligned} & \frac{M}{U}U = \nu E \\ \Leftrightarrow \quad & \frac{M}{U}\overline{L} - \frac{M}{U}E = \nu E \\ \Leftrightarrow \quad & p(\theta)\overline{L} = (\nu + p(\theta))\, E. \end{aligned}$$

Therefore, $e(\theta) := \frac{E}{\overline{L}} = \frac{p(\theta)}{\nu + p(\theta)}$ follows. ■

**Proposition 9** *Using matching function (2), production function (9'), the change in employment (11), capital in efficiency units (20'), the optimal condition for imports (22"), net wealth definition (23), the wealth market equilibrium (28), the saving hypothesis (29), budget constraint (30) and the steady-state condition for the wealth market (34), the balanced accumulation function (35) is implied.*

**Proof.** Rewrite (30) as

$$\begin{aligned} & Y = X - p_z Z - c_v V + rB \\ \Leftrightarrow \quad & sY = s\left[X - p_z Z - c_v V + rB\right]. \end{aligned}$$

Use (28) and (29) in efficiency units and substitute it in the above equation, we get

$$\dot{k} + \dot{b} = s\left[x - p_z z - c_v v + rb\right]. \tag{30'}$$

Define vacancies in efficiency units as $v := V/\lambda E$ and use (2) and (11), then

$$v = \frac{\left(\hat{E} + \nu\right)\hat{\lambda}}{\lambda}\theta^{\beta}. \tag{41}$$

Substitute this as well as (9'), (22") in (30') to get

$$\dot{k} + \dot{b} = s\left[(1-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} - \frac{c_{v0}\hat{\lambda}(\hat{E}+\nu)\theta^{\beta}}{\lambda_0} + rb\right] \tag{30"}$$

Now, substitute (30") in the steady-state condition (34) and get

$$\begin{aligned} \dot{a} &= \dot{k} + \dot{b} - (\hat{\lambda} + \hat{E})a = 0 \\ \dot{a} &= s\left[(1-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} - \frac{c_{v0}\hat{\lambda}(\hat{E}+\nu)\theta^{\beta}}{\lambda_0} + rb\right] - (\hat{\lambda} + \hat{E})a = 0. \end{aligned}$$

Use for $b$ equation (23) in efficiency units, for $k$ equation (20') and $\dot{k} = 0$[32], then

$$\dot{a} = s\left[(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} - \frac{c_{v0}\hat{\lambda}(\hat{E}+\nu)\theta^{\beta}}{\lambda_0}\right] + \left(sr - \hat{\lambda} - \hat{E}\right)a = 0 \tag{30"'}$$

follows. Now use $\dot{a} = 0$ and $\hat{E} = 0$, the balanced wealth accumulation function

$$\theta^{\beta} = \frac{\lambda_0}{c_{v0}\hat{\lambda}\nu}\left\{(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} + \left(sr - \hat{\lambda}\right)a\right\} := \Phi(a)$$

is implied. ■

[32] $\dot{k} = 0$ is valid, since the capital stock in efficiency units is for a given world interest rate constant.

**Proposition 10** *Let $sr-\hat{\lambda}<0$, the balanced asset accumulation function is a linear function with $\Phi(0)>0$, $\Phi(\infty)=\infty$, $\Phi'(a)<0$ and $\Phi''(a)=0$.*

**Proof.** $\Phi(0):=\frac{\lambda_0}{c_{v0}\hat{\lambda}s\nu}(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}>0$, $\Phi(\infty)=\infty$. Let $sr-\hat{\lambda}<0$, then

$$\Phi'(a)=\frac{\lambda_0}{c_{v0}\hat{\lambda}\nu}(sr-\hat{\lambda})<0$$

and $\Phi''(a)=0$. ■

**Proposition 11** *A steady state exists and is unique with*

$$\widetilde{a}\begin{cases}\geq 0\\ <0\end{cases} \quad for \quad \begin{cases}\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{[\nu-\hat{\lambda}+r]}+\alpha+\gamma-1\geq 0\\ \frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{[\nu-\hat{\lambda}+r]}+\alpha+\gamma-1<0\end{cases}.$$

**Proof.** Define $\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\lambda_o}{\widehat{\lambda}c_{v0}\left[\nu-\widehat{\lambda}+r\right]}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}:=T$, $\frac{\lambda_0}{c_{v0}\hat{\lambda}\nu}:=D$ and $\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{1-\gamma}{\alpha+\gamma-1}}:=M$, $\frac{r}{\alpha}(1-\alpha-\gamma):=G$ and use

$$\begin{aligned}\Psi(a)&=T\\ \Phi(a)&=D\left\{MG+(sr-\hat{\lambda})a\right\},\end{aligned}$$

then

$$T=D\left[MG+(sr-\hat{\lambda})a\right],$$

respectively

$$\begin{aligned}\widetilde{a}&=\frac{1}{sr-\hat{\lambda}}\left(\frac{T}{D}-MG\right)\\ &=\underbrace{\frac{1}{sr-\hat{\lambda}}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}}_{>0}\\ &\quad\underbrace{\left\{\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{\left[\nu-\widehat{\lambda}+r\right]}+\alpha+\gamma-1\right\}}_{\gtrless 0}\\ &\gtrless 0.\end{aligned}$$

Analyzing the last term yields

$$\underbrace{\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{\left[\nu-\widehat{\lambda}+r\right]}}_{>0}+\underbrace{\alpha+\gamma}_{>0}-1\gtrless 0,$$

therefore,

$$\widetilde{a}\begin{cases}\geq 0\\ <0\end{cases} \quad for \quad \begin{cases}\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{[\nu-\hat{\lambda}+r]}+\alpha+\gamma-1\geq 0\\ \frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{[\nu-\hat{\lambda}+r]}+\alpha+\gamma-1<0\end{cases}. \quad ■$$

**Proposition 12** *The domestic country is net creditor, i.e. $\widetilde{a}\geq 0$, if $\frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\nu}{[\nu-\hat{\lambda}+r]}$ $+\alpha+\gamma-1\geq 0$.*

**Proof.** See above. ■

**Proposition 13** *If the domestic country is a net debtor (creditor), its current account shows a constant deficit (surplus) in the overall steady state, i.e.* $\widetilde{ca} = \hat{\lambda}\widetilde{a} < 0$.

**Proof.** Define the current account (25) and the balance of payments (26) in efficiency units, i.e.

$$\begin{aligned} ca &= ex - p_z z - ra \\ 0 &= ex - p_z z - ra - \dot{b} \end{aligned}$$

and, therefore,

$$ca = \dot{b}. \tag{26'}$$

Substituting (26') in the steady-state condition for the wealth market (34) and $\dot{k} = 0$,

$$\widetilde{ca} = \hat{\lambda}\widetilde{a}$$

is implied. Therefore, if $\widetilde{a} < 0$, then $\widetilde{ca} < 0$ (and vice versa). ■

**Proposition 14** *The dynamic factor allocation function is given by* $\dot{\theta} = \frac{\theta}{\beta}\left\{r + \nu - \hat{\lambda}\right.$ $\left. - \frac{(1-\alpha-\gamma)(1-\omega)(1-\beta)\lambda_0}{c_{v0}\hat{\lambda}\theta^\beta}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}\right\}$.

**Proof.** Substitute (20') in (21"), then

$$\theta^\beta = \frac{(1-\alpha-\gamma)(1-\beta)(1-\omega)\lambda_o}{\widehat{\lambda} c_{v0}\left[r + \nu - \widehat{\lambda} + \beta(\hat{U} - \hat{V})\right]}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}.$$

Use $\hat{U} - \hat{V} = -\hat{\theta}$, i.e. differentiate $\theta := V/U$ with respect to time, then the dynamic factor allocation function

$$\dot{\theta} = \frac{\theta}{\beta}\left\{r + \nu - \hat{\lambda} - \frac{(1-\alpha-\gamma)(1-\omega)(1-\beta)\lambda_0}{c_{v0}\hat{\lambda}\theta^\beta}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}}\right\}$$

is implied. ■

**Proposition 15** *The dynamic asset accumulation function can be written as*

$$\begin{aligned} \dot{a} &= s\left[(1-\alpha-\gamma)\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} - \frac{c_{v0}\hat{\lambda}(\hat{E}+\nu)\theta^\beta}{\lambda_0}\right] \\ &\quad + \left(sr - \hat{\lambda} - \hat{E}\right)a. \end{aligned}$$

**Proof.** See derivation of equation (30"'). ■

**Proposition 16** *Suppose* $h_\theta = \frac{r+\nu-\hat{\lambda}}{\beta} - \frac{(1-\beta)^2(1-\alpha-\gamma)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\beta q(\theta)}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} > 0$, $|H|$ *is negative, i.e. the unique steady-state is a saddle point.*

**Proof.** Using $h_\theta = \frac{r+\nu-\hat{\lambda}}{\beta} - \frac{(1-\beta)^2(1-\alpha-\gamma)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\beta q(\theta)}\left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} > 0$, the signs of the coefficient matrix of $H$ can be characterized by

$$H = \begin{bmatrix} 0 & + \\ + & + \end{bmatrix}$$

and, therefore,

$$
\begin{aligned}
|H| &= 0 - \left[\hat{\lambda} + \hat{E} + sr\right] \left\{ \frac{r + \nu - \hat{\lambda}}{\beta} \right. \\
&\qquad \left. - \frac{(1-\beta)^2 (1-\alpha-\gamma)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\beta q(\theta)} \left(\frac{\gamma}{p_z}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\gamma-1}} \right\} \\
&< 0
\end{aligned}
$$

is implied. That is one characteristic root is positive and the other is negative, implying that the unique steady-state is a saddle point. ■

**Development of Long-Term Unemployment in Industrialized Countries**

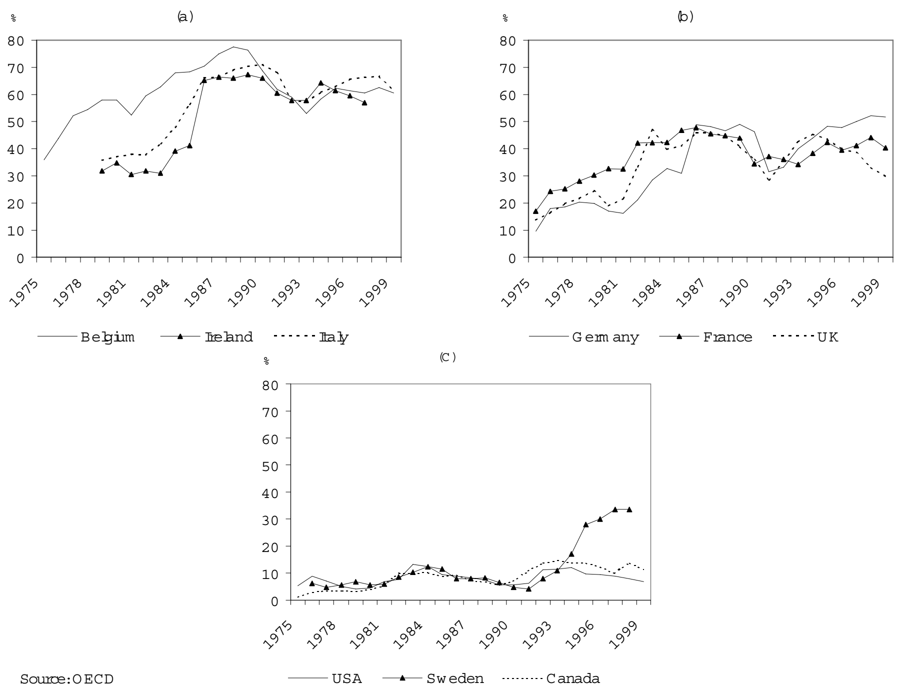


Figure 7: *Development of Long-Term Unemployment on Total Unemployment for Groups of Countries.*

Figure 7 shows the increase in long-term unemployment for selected groups of countries. Figure 7a states a group of countries characterized by high shares of long-term unemployment. In 1975, Belgium had 36 per cent long-term unemployment; this share increased until the end of the 1990s to over 60 per cent. Italy and Ireland had nearly 67 and 57 per cent, respectively, long-term unemployment in 1999. This group had an average growth rate of long-term unemployment of about 2 per cent annually.

The countries, shown in Figure 7b, are characterized by medium levels and higher average growth rates of long-term unemployment. The share of long-term unemployment increases in Germany from 10 per cent in 1975 up to 50 per cent in 1999. France and the U.K. show nearly the same structure: their proportions increased from 17 per cent in 1975 to up to 40 per cent.

A third country group with relatively low levels, but also relatively high growth rates of long-term unemployment can be identified in Figure 7c. Canada started with 1 per cent long-term unemployment and had nearly 11 per cent in 1999; Sweden had around 6 per cent and ended up with 33 per cent. In the U.S., the proportion of long-term unemployed workers is over the whole period almost constant at about 6 per cent and the average growth rate is constant as well. However, Sweden and Canada display annual average growth rates of 7 respectively 9 per cent.