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Abstract

How does technical progress affect long-term unemployment? The relationship between long-term unemployment and the rate of growth attributable to technical progress is evaluated in a growth-matching-model with heterogeneous jobless workers and with endogenously determined long-term unemployed resulting from skill-depreciation. For innovation economies characterized by high steady-state levels of capital intensities the model shows that, due to a capitalization effect and a qualification-mismatch effect, increasing technological progress has adverse implications for long-term unemployment. Furthermore, for imitation economies with low steady-state capital intensities increasing technological progress can be either favorable or less favorable for long-term unemployment depending on whether the creative destruction effect or the capitalization effect dominates.

Zusammenfassung

Wie beeinflusst der technische Fortschritt die Langzeitarbeitslosigkeit? Die Beziehung zwischen Langzeitarbeitslosigkeit und der Wachstumsrate des technischen Fortschritts wird in einem Wachstums-Matching-Modell mit heterogenen Arbeitslosen und mit endogen bestimmter Langzeitarbeitslosigkeit, die auf Qualifikationsverlust zurückzuführen ist, analysiert. Für Innovationsländer, die durch hohe gleichgewichtige Kapitalintensitäten charakterisiert werden können, wird in dem Model gezeigt, dass steigender technischer Fortschritt aufgrund eines Kapitalisierungs- und eines Qualifikations-Mismatch-Effekts negative Implikationen für die Langzeitarbeitslosigkeit hat. Für Imitationsländer mit einer geringen gleichgewichtigen Kapitalintensität kann beschleunigter technischer Fortschritt positiv oder negativ auf die Langzeitarbeitslosigkeit wirken, je nachdem, ob der Effekt der kreativen Zerstörung oder der Kapitalisierungseffekt dominiert.

JEL Classification: E 24, J41, O41

Keywords: long-term unemployment, mismatch, growth, search, matching

1 Introduction

Data on long-term unemployment[1] show a huge increase in the level and the growth rates of long-term unemployment in industrialized countries and simultaneously these countries display positive GNP-growth rates per capita. A natural question then becomes, How does technical progress affect long-term unemployment?

Stylized Facts

Figure 1a shows a group of countries characterized by high shares of long-term unemployment. In 1975 Belgium displays 36 per cent long-term unemployment of total unemployment; this share increases until 1999 up to over 60 per cent. Italy and Ireland have nearly 67 respectively 57 per cent long-term unemployment in the end of the 90s. In this group the average growth rate of long-term unemployment is at about 2 per cent.

The countries shown in Figure 1b are characterized by medium levels and higher average growth rates of long-term unemployment. The share of long-term unemployment increases in Germany from 10 per cent in 1975 up to 50 per cent in 1999. France and the U.K. show nearly the same structure: their proportions rise from 17 per cent in 1975 up to 40 per cent at the end of the last decade.

A third country group with relatively low levels but relatively high growth rates of long-term unemployment can be identified in Figure 1c. Canada starts with 1 per cent long-term unemployment and this increases up to nearly 11 per cent in 1999; Sweden starts with 6 per cent and ends up with 33 per cent. In the US the proportion of long-term unemployed workers is over the whole period almost constant at about 6 per cent and the average growth rate is constant as well. However, Sweden and Canada display annual average growth rates of 7 respectively 9 per cent.

In Figure 2 GNP per capita growth rates for the groups of countries are shown. Ireland displays the highest average annual growth rate of 3.7 per cent followed by Italy with 1.9 per cent and Belgium and the U.K. with 1.8 per cent. All other countries have positive growth rates at about 1 per cent or higher.

Thus, the stylized facts show that long-term unemployment is a serious problem and simultaneously industrialized countries have positive growth rates.

Regarding this stylized fact, the relationship between long-term unemployment and the rate of growth attributable to technical progress at different steady-state

[1] Long-term unemployment is defined as percentage on total unemployment.

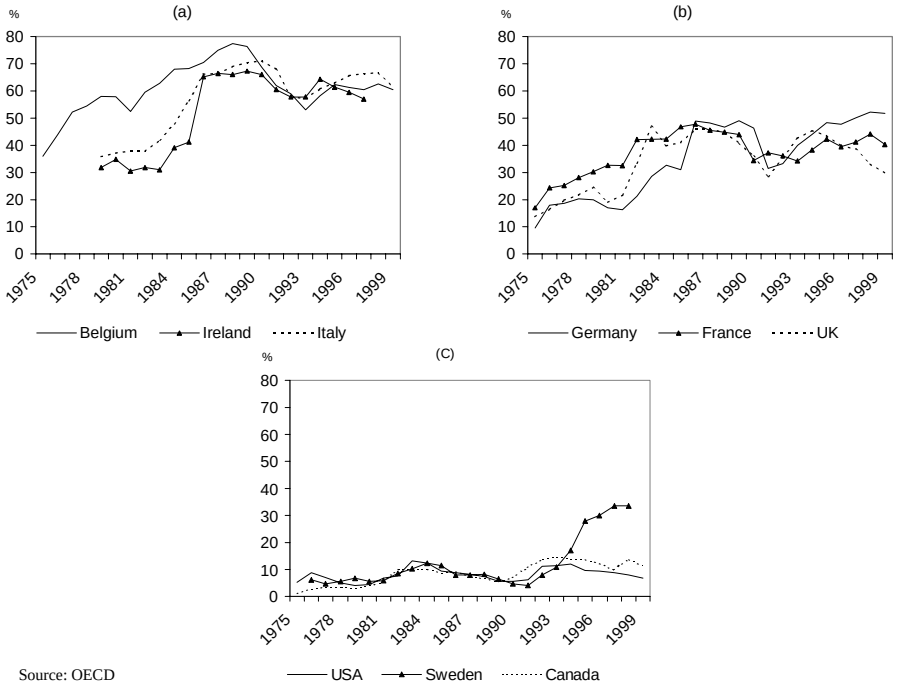

Figure 1: *Development of Long-Term Unemployment on Total Unemployment for Groups of Countries.*

levels of capital intensities is evaluated in a growth-matching-model with a heterogeneous unemployment pool, consisting of short-term and long-term unemployed, and with endogenously determined skill-depreciation of the long-term jobless workers. It will be shown that, due to a capitalization effect and a qualification-mismatch effect, increasing technological progress has adverse implications on long-term unemployment in innovation economies which are characterized by high steady-state levels of capital intensities. Furthermore, for imitation economies with low steady-state capital intensities technological progress can be favorable or unfavorable for long-term unemployment depending on whether the creative destruction effect or the capitalization effect dominates.

Before the model is developed in section 3, a short review of the literature discussing the relationship between technical progress and unemployment is given in the following section. Section 4 analyses the steady-state solution and the stability of the model and, thereafter, economic implications are shown in section 5. Section 6 concludes the discussion.

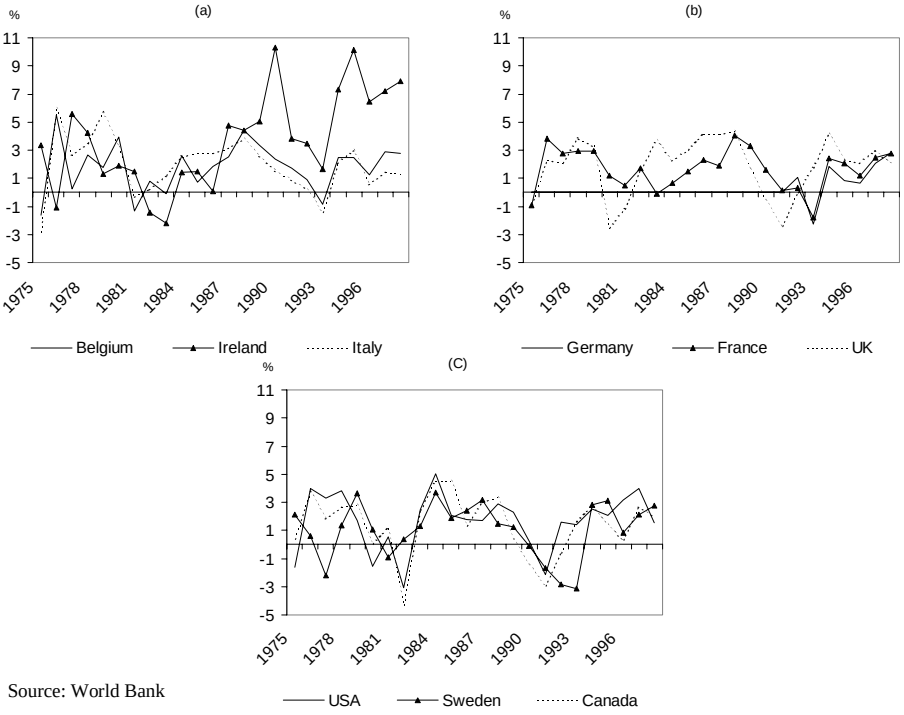

Figure 2: *GNP Per Capita Growth for Country Groups.*

2 Review of Literature

Recently, expanding literature focuses, on the one hand, mainly on unemployment and technical progress – usually in growth-matching models that neglect the influence of different steady-state capital intensity levels on unemployment – and, on the other hand, the huge increase in long-term unemployment is predominantly discussed in more empirical related approaches[2] explaining the reasons and implications of reduced outflow rates on long-term unemployment.[3]

The more theoretical discussion on technical progress and unemployment does not determine any clear-cut relationship. Pissarides (1990) shows that an increase in the growth rate of technical progress results in higher revenues accruing from the successful filling of vacancies and firms offer additional vacancies in the following periods. Therefore, a positive link between employment and productivity

[2]Exceptions are Coles, Masters (2000), Ljungvist, Sargent (1998, 1995), Acemoglu (1995), Blanchard, Diamond (1994), Pissarides (1992) and Lockwood (1991).

[3]See for example Gora, Schmidt (1998), Boeri, Wörgötter (1998), Rosen (1997), Burgess (1994), Layard, Nickell, Jackman (1991) and Jones, Manning (1992).

growth equally distributed among all sectors and all jobs is indicated. According to Aghion, Howitt (1994) technical progress is not equally distributed implying that technical progress generates and destroys jobs at the same time. This causes a creative destruction effect, which generates that an increase of technical progress induces via the net destruction of jobs unemployment, and a capitalization effect representing that an increase in technical progress leads to more employment by the net creation of jobs. Aghion, Howitt shows that, at small rates of productivity growth the creative destruction effect dominates the capitalization effect and a reduction in employment is induced; whereas at high rates of technological progress the opposite holds causing an increase in employment. Thus, a hump shaped relationship between productivity growth and unemployment is implied.
While in both approaches productivity growth generates unemployment, Bean, Pissarides (1993) point to the other direction of causation requiring that unemployment determines technological progress. Using this hypothesis they attain a positive respectively negative interdependence between growth and unemployment depending on whether the point of view is Classical or Keynesian. Postel-Vinay (1998) and Mortensen, Pissarides (1998) confirm the ambiguous link between growth and unemployment in endogenous growth-matching-models. They show in stochastic matching models with heterogenous productivities that, via the creation of new jobs, increasing productivity growth leads to an increase in employment when renovation costs are low and it induces unemployment when renovation costs are high.

Some recent studies analyze the effects of *skill-biased* technological shocks on unemployment respectively long-term unemployment. Coles, Masters (2000) examine in a search-matching model the effect of skill depreciation on the equilibrium level of unemployment when long-term unemployment emerges as endogenous phenomenon. Their model implies that today's recession, which leads to longer unemployment spells, impacts on the distribution of market skill levels in the future. Due to this result, they conclude that subsidizing retraining to reduce long-term unemployment is inappropriate; a better way is to subsidize vacancy creation. The argument of Ljungqvist, Sargent (1998) points in the same direction and identifies generous welfare schemes as obstacles for reducing long-term unemployment. Their point is that generous welfare schemes tend to be more prone to generate high levels of unemployment when economies undergo rapid structural change. If redundancy is associated with the loss of skills, abundant welfare payments make

unemployed more reluctant to take up poorly paid jobs in start-up industries. Thus, an increase in long-term unemployment is implied, and increasing structural change, interpreted as the rate of skill decay, will always raise unemployment. This point of view is confirmed by MARIMON, ZILIBOTTI (1999). They show in a search-matching model with heterogeneity in productivities that differences in unemployment insurance result in differences in unemployment and productivity growth, when countries experience a skill-biased technological shock. Due to the complementarity between capital and capital-specific-skills, the mismatch in the economy is enlarged. MORTENSEN, PISSARIDES (1999) also examine the effects of skill-biased shocks that increase the spread of productivities across different skills. These skill-biased shocks are modelled as changes in the complementarity between new capital and labor, favoring the more skilled and eroding the productivity of the less skilled. Therefore, pure skill-biased shocks lead to higher mean unemployment and to longer unemployment durations.

3 The Economy

As this discussion shows, in growing economies the skill-biased mismatch respectively qualification-mismatch is a crucial determinant for the existence and the increase of long-term unemployment. In our approach, it is defined as the mismatch between the human capital of unemployed and the job-specifications demanded by firms, and finds its expression especially with increasing technological progress. The higher the growth rate of technological progress, i.e. the faster new production technologies are introduced, the more qualification-intensive and specific the job-requirements are that arise from the creation of new jobs. Therefore, new technologies need specific human capital being capable of using new machines, computers etc. Economies with high steady-state capital intensities are characterized by rapid innovation development and rapid introduction of new technologies. Therefore, these countries are innovation rather than imitation economies. If a constant labor force and a heterogeneous unemployment pool, consisting of short-term and long-term unemployed, is regarded and if on-the-job-search is omitted, only jobless workers can fill new vacancies resulting from the innovation of new technologies. Furthermore, if the qualification level of the unemployed does not grow with the same rate as technical progress, i.e. the rate of technological progress is strictly positive and the accumulation rate of human capital is zero, human capital of the unemployed

depreciates with technical progress and it depreciates the faster, the longer the unemployment duration takes. Due to this skill-depreciation, firms are reluctant to hire long-term unemployed implying that only short-term unemployed are matched with new vacancies. Therefore, long-term unemployment emerges as endogenous phenomenon and it increases as qualification-mismatch rises.

These implications are derived in a growth-matching-model with the labor market characterized by matching-frictions and capital accumulation is described by a neoclassical growth process.[4] Matching-frictions represent the search process needed to fill vacancies and the search process is modelled as taking place between job vacancies and unemployed workers.[5] Even in equilibrium, which is defined as a flow equilibrium, i.e. inflows are equal to outflows, the labor market is marked by search or matching-frictions. If no frictions were present, laid-off workers would find immediately new jobs and equilibrium unemployment would not exist. The existence of frictions implies further that outflows depend on the labor market tightness inducing that the matching-probability is influenced by the levels of unemployment and vacancies. Therefore, each trading partner faces market externalities determined by the number of traders on each side of the market.[6]

Furthermore, due to matching-frictions, trading partners have some monopoly power and successful matching yields additional profits which are shared between firms and workers. The division of profits can be modelled by a Nash bargaining approach or simply by sharing the additional produced marginal product with the sharing proportions are determined by the bargaining power of the trading partners.[7] It is assumed that all job-workers pair are equally productive. Wages are then determined by the sharing rule.

The unemployment pool consists of heterogeneous unemployed workers and the fraction of long-term unemployment is determined by the duration of unemployment itself and by the rate of technical progress. The positive dependence between the average duration of unemployment and the fraction of long-term unemployed can be justified by the extreme skill-depreciation and by the motivation losses of the

[4] The model is similar to that of Pissarides (1990). For an endogenous growth model with matching see for example Postel-Vinay (1998) and for a RBC-model see for example Merz (1995, 1999).

[5] See also Blanchard, Diamond (1994, 1989).

[6] See also Merz (1995) and Feve, Langot (1996).

[7] See also Gries, Jungblut, Meyer (1997 a, b).

long-term unemployed during their jobless time.[8] If the average, endogenously determined duration of unemployment increases, increased long-term unemployment is implied. Furthermore, increasing technical progress induces rising long-term unemployment, since long-term unemployed do not possess the know-how and the abilities to handle the latest production methods.

For attaining the steady-state solution and the determinants of the equilibrium level of long-term unemployment, an efficient factor allocation function and a balanced accumulation function are derived. The first mentioned function which is implied by the intertemporal demand decisions of firms characterizes labor market structures and describes the optimal factor allocations in the labor market. As long as the labor market has not reached the long-run equilibrium, structures will change permanently. This is reflected by differences in inflows and outflows and by a permanently changing level of long-term unemployment. The second mentioned function represents the steady-state of the goods market and characterizes the growth process of the economy. In long-run equilibrium all relevant variables grow with the same rate and the vacancy level as well as the share of long-term unemployment have reached long-run positions.

The Labor Market

The aggregate labor endowment of households is constant and denoted by $L = \overline{L}$. At any time labor is either employed or unemployed; the employed workers are denoted as E and the unemployed as U. Thus, the labor force is represented by

$$\overline{L} = E + U. \tag{1}$$

The labor market is characterized by search frictions with firms looking for jobless workers filling vacancies and unemployed searching for a job. Both sides of the market have incomplete information about the opposite market side. The level of search activities is represented by the number of vacancies V, the number of unemployed U and the number of matches M formed at any point in time. Furthermore, since newly created vacancies are endowed with the most recent technology, the number of matches is also determined by the rate of technological progress $\widehat{\lambda}$ representing the diffusion of technological know-how. If an economy has a high rate of technological progress, only few unemployed workers can fill the vacancies and the number of matches is relatively low. In order to represent this, it is assumed

[8] See Layard (1997).

that technological knowledge of the unemployed does not grow with the same rate as technological progress. The underlying matching technology is defined as

$$M = m(V, U; \widehat{\lambda}) = V^{1-\beta} U^{\beta} \widehat{\lambda}^{-1}, \tag{2}$$

with β as the search intensity of the average unemployed worker and the matching function is assumed to be homogeneous of degree one. Furthermore, the indicator for labor market tightness is denoted by the ratio of vacancies to unemployed $\theta = V/U$ and

$$p(\theta) := M/U = m(V/U, 1; \widehat{\lambda}), \quad p_\theta > 0 \tag{3}$$

is the matching-probability for the unemployed and

$$q(\theta) := M/V = m(1, U/V; \widehat{\lambda}), \quad q_\theta < 0 \tag{4}$$

is the probability of filling vacancies. Both probabilities depend on labor market tightness and reflect the externalities each trading partner faces. If the number of jobless workers increases, the matching-probability for the average unemployed will decrease and simultaneously the probability of filling vacancies will increase.
Due to constant returns of scale, the average duration in unemployment is defined as

$$\rho(\theta) := U/M, \quad \rho_\theta < 0 \tag{5}$$

and it rises when the labor market becomes tighter which is characterized by increasing unemployment for given vacancies.

Furthermore, two types of jobless workers are distinguished: short-term and long-term unemployed, U^S respectively U^L, and the heterogeneous unemployment pool is defined as

$$\begin{aligned} U &= U^S + U^L \\ U &= [1 - \phi(\rho; \widehat{\lambda})]U + \phi(\rho; \widehat{\lambda})U, \quad 0 < \phi < 1, \;\; \phi_\rho, \phi_{\widehat{\lambda}} > 0, \end{aligned}$$

with $\phi(\rho; \widehat{\lambda})U$ as the long-term unemployed. The long-term jobless workers show significant different search behavior than short-term unemployed. They are looking for new jobs with less search intensity and, due to the long unemployment duration, they are demoralized and discouraged.[9] During their jobless time their human capital is exposed to large depreciation losses and, since they are not trained and do

[9] See also Layard, Nickell, Jackman (1991).

not accumulate any additional knowledge, i.e. without allocating any resources to the long-term unemployed, they are not able to handle the latest production technologies. Therefore, the number of long-term jobless workers depends positively on the unemployment duration ρ and positively on the rate of technical progress $\widehat{\lambda}$.

If new job-matches are formed, each match generates additional revenues and, because both trading partners have monopoly power,[10] unemployed workers and firms could bargain over the additional produced profits; or the profits are simply shared using a sharing rule. This sharing rule determines the profit proportion, the new workers get and therefore, the wage results as a constant fraction of the marginal product

$$w = \omega F_E(k), \quad 0 < \omega < 1, \tag{6}$$

with ω denoting the sharing proportion and it represents the monopoly power of unemployed workers.

The Goods Market

Each firm uses capital K, labor L and the current state of technological progress $\lambda := \lambda_0 e^{\hat{\lambda} t}$ to produce a homogenous good X. Production is described by a Cobb-Douglas-function:

$$X = F(K, \lambda E) := K^{\alpha}[\lambda E]^{1-\alpha} \tag{7a}$$

$$\Leftrightarrow \quad x = k^{\alpha} \tag{7b}$$

with $x := X/\lambda E$ and $k := K/\lambda E$.

For the representative firm demand decisions concern changes in real capital and in employment. It is supposed that installation costs of $c_I I$ [with $0 < c_I < 1$] arise with c_I as the fraction of installation costs used for investments I.
The change in employment is determined by inflows in and outflows out of unemployment. The inflows into unemployment are characterized by the separation of existing job-matches at any point in time and are described by the exogenously given separation rate ν times the workers E. Thus, inflows characterize the number of unproductive jobs which generate layoffs.[11] On the other hand, outflows

[10] See also Nickell (1999) and Zanchi (2000) for a recent discussion of the wage determination in search models.

[11] For an exogenous separation rate see also Pissarides (1990) and Postel-Vinay (1998) and for an endogenous rate see Mortensen/Pissarides (1994, 1998).

are represented by the flow of newly formed job-matches and, therefore, by the matching-function $m(U, V; \hat{\lambda})$. Firms create and offer new productive jobs and they have to fill these vacancies by searching for suitable workers. At the aggregate level, the filling of vacancies depends on the number of unemployed, the number of offered vacancies, the search intensities of firms and unemployed and the rate of technical progress; all determinants are expressed in the matching-function. Taking outflow and inflow together, the dynamics of employment result as the difference between outflows and inflows and can be expressed as

$$\dot{E} = m(U, V; \hat{\lambda}) - \nu E. \tag{8}$$

Each vacancy induces search costs of c_v with $c_v := c_{v0} e^{\hat{\lambda} t}$. Since the newest jobs contain the latest technology, it is costly for the firm to find unemployed workers being able to handle most recent technologies. Therefore, search costs grow with the rate of technical progress.

Taking these aspects into consideration, the representative firm faces the following intertemporal optimization problem with the current flow of profits as output minus factor payments minus search expenditures. Denoting r as the discount factor the firms maximization can be written as

$$\max_{I,V} \int_0^\infty \{F(K, \lambda E) - rK - wE - c_I I - c_v V\} e^{-rt} dt$$

$$\begin{aligned} \text{s.t.} \quad & \dot{E} = m(U, V; \hat{\lambda}) - E \\ & \dot{K} = I \\ & K(0), E(0), V(0), U(0) \quad \text{given.} \end{aligned}$$

In order to solve the optimization problem, a present-value Hamiltonian function $\mathcal{H}(K, E, V, I, \mu_{1,} \mu_2)$ with costate variables μ_i $[i = 1, 2]$ is set up. Denoting F_j as the

partial derivative of $F(\cdot)$ with respect to $j = K, E$, the Hamiltonian conditions are

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial V} &= 0 \quad \Longleftrightarrow \quad -e^{-rt}c_v + \mu_1 \frac{\partial m}{\partial V} = 0 && (9)\\
-\dot{\mu}_1 &= \frac{\partial \mathcal{H}}{\partial E} \quad \Longleftrightarrow \quad -\dot{\mu}_1 = e^{-rt}[F_E - w] - \mu_1 \nu && (10)\\
\dot{E} &= \frac{\partial \mathcal{H}}{\partial \mu_1} \quad \Longleftrightarrow \quad \dot{E} = m(U, V) - \nu E && (11)\\
\frac{\partial \mathcal{H}}{\partial I} &= 0 \quad \Longleftrightarrow \quad -e^{-rt}c_I + \mu_2 = 0 && (12)\\
-\dot{\mu}_2 &= \frac{\partial \mathcal{H}}{\partial K} \quad \Longleftrightarrow \quad -\dot{\mu}_2 = e^{-rt}[F_K - r] && (13)\\
\dot{K} &= \frac{\partial \mathcal{H}}{\partial \mu_2} \quad \Longleftrightarrow \quad \dot{K} = I && (14)
\end{aligned}$$

with the transversality condition

$$\lim_{t \to \infty} \mathcal{H}(t) = 0.$$

The first order condition for capital respectively labor are given by

$$\begin{aligned}
F_K(k) &= (1 + c_I)\, r && (15)\\
F_E(k) &= w + \frac{\hat{\lambda}}{1 - \beta} c_v \left[r - \hat{\lambda} + \beta \left(\hat{U} - \hat{V} \right) + \nu \right] \theta^{\beta} && (16)
\end{aligned}$$

with $F_j(k)$ $[j = K, E]$ as marginal products and the right hand sides are marginal costs of capital respectively labor.

After describing the intertemporal optimization problem of the representative firm, the model has to be closed by denoting aggregate income and the budget constraint. Factor income of the households Y is defined as the remuneration of production factors capital and labor

$$Y := rK + wE \tag{17}$$

with the wage rate w.

The output is used for factor income Y, installation costs $c_I I$ and search costs $c_v V$:

$$X = Y + c_I I + c_v V. \tag{18}$$

Both of the last terms represent the profit income of firms that is completely used for installation and search costs $(1 - \omega) F_E(k) = c_I I + c_v V$.

Equilibrium of the Goods Market

In the closed economy households consume and save a constant fraction of their income and the equilibrium for the goods market is characterized by

$$I = S = sY \tag{19}$$

with S as savings and s as the saving rate.

4 Steady-State Solution

Analyzing the steady-state solution, the long-run equilibrium for the labor market and the steady-state for the goods market are derived separately and can be characterized by a efficient factor allocation function respectively a balanced accumulation function.

Steady-State of the Labor Market

The steady-state of the labor market is deduced by using the flow condition of the labor market. This condition requires that the inflows are equal to the outflows and, therefore, the change in employment is zero:

$$\dot{E} = 0 \;\Leftrightarrow\; V^{1-\beta}U^{\beta}\hat{\lambda}^{-1} = \nu E. \tag{20}$$

Furthermore, due to neglecting on-the-job-search, the flow of new created vacancies is identical to the employment flow, i.e. $\dot{V} = \dot{E} = 0$, and because of a constant labor force, the employment and unemployment levels are constant in the long-run equilibrium, i.e. $\dot{E} = -\dot{U} = 0$. These conditions imply that steady-state labor market tightness is also constant, i.e. $\dot{\theta} = 0$, and that the steady-state growth rates of unemployment and vacancies are zero, i.e. $\hat{V} = \hat{U} = 0$.

Using these conditions, the *efficient factor allocation function* for the stationary labor market can be derived:[12]

$$\theta^{\beta} = \frac{\lambda_0(1-\alpha)(1-\beta)(1-\omega)}{c_{v0}\hat{\lambda}\left[\left(\frac{\alpha}{1+c_I}\right)k^{\alpha-1} + \nu - \hat{\lambda}\right]}k^{a} =: \Psi_1(k). \tag{21}$$

It shows all combinations of capital intensity and labor market tightness that reflect the long-run equilibrium of the labor market. The steady-state of the labor-market

[12] For the detailed derivation of the efficient factor allocation function see appendix.

is influenced by several exogenous variables and it will change when the exogenous environment changes. In the (θ^{β}, k) plane it has a positive concave shape.[13]

Furthermore, in the long-run labor market equilibrium the steady-state employment rate is given by[14]

$$e(\theta) := \frac{E}{\bar{L}} = \frac{p(\theta)}{\nu + p(\theta)}, \quad e_{\theta} > 0. \tag{22}$$

Therefore, the employment probability depends positively on labor market tightness θ and on the matching-probability $p(\theta)$ and negatively on the separation rate ν. The higher the separation rate, the lower the steady-state employment rate. Furthermore, the steady-state unemployment rate is determined as well as

$$1 = e(\theta) + u(\theta),$$

where the steady-state unemployment rate $u(\theta)$ is defined as $u(\theta) := U/\bar{L}$.

Thus, the steady-state for the labor market is described by an efficient factor allocation function that defines all equilibrium combinations of labor market tightness and capital intensity.

Steady-State of the Goods Market

As common in neoclassical growth models, the long-run steady-state is characterized by a constant capital intensity, i.e.

$$\dot{k} = 0 \tag{23}$$

The steady-state of the goods market can be described by a *balanced capital accumulation function*:[15]

$$\theta^{\beta} = \frac{\lambda_0}{c_{v0}\hat{\lambda}\nu}\left[k^{\alpha} - \frac{(1 + c_I s)\,\hat{\lambda}}{s}k\right] =: \Phi_1(k) \tag{24}$$

This function shows all combinations of labor market tightness and capital intensity characterizing the steady-state in the goods market.

Furthermore, in the (θ^{β}, k) plane the balanced accumulation function has – until the maximum is reached – a positive slope, in the maximum a slope of zero and behind the maximum a negative slope.[16]

[13] See appendix.

[14] See appendix.

[15] For a detailed derivation of the balanced accumulation fuction see the appendix.

[16] See appendix.

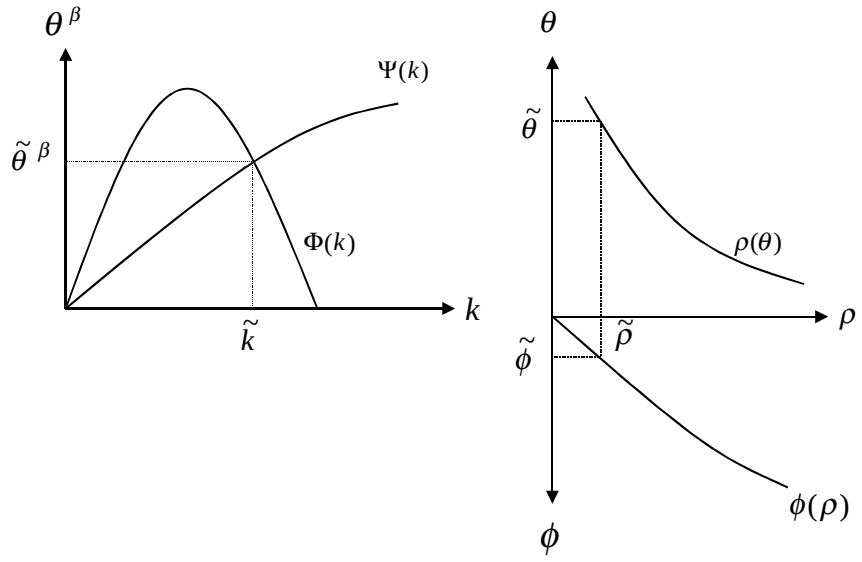

Figure 3: *Steady-State of the Economy.*

After deriving the equilibrium conditions for the steady-state labor market respectively for the steady-state goods market separately, both determine together the overall steady-state, i.e. the efficient factor allocation function and the balance capital accumulation function simultaneously define the steady-state values for θ and k. In Figure 3 the steady-state search equilibrium $(\tilde{\theta}^{\beta}, \tilde{k})$ is graphed at the intersection of both functions. Due to the shape of both functions the steady-state exists and is unique.

Once the steady-state search equilibrium $(\tilde{\theta}^{\beta}, \tilde{k})$ is determined, the steady-state values for the matching probability $\tilde{p}$, the steady-state employment respectively unemployment rate $\tilde{e}$ respectively $\tilde{u}$ can be derived. The steady-state employment and unemployment levels are fixed as well: $\tilde{E} = e(\tilde{\theta})\bar{L}$ and $\tilde{U} = u(\tilde{\theta})\bar{L}$. Furthermore, steady-state labor market tightness determines the equilibrium unemployment duration $\tilde{\rho}$ and the steady-state fraction of the long-term unemployed $\tilde{\phi}$ (see Figure 3).

Beside the determination of the steady-state labor market variables, the growth and accumulation process is fixed. In the long-run equilibrium the capital stock, the production and income levels grow with the rate of technical progress, i.e. $\hat{K} = \hat{X} = \hat{Y} = \hat{\lambda}$.

Stability of the Steady-State

The transitional behavior of the labor market tightness is characterized by the

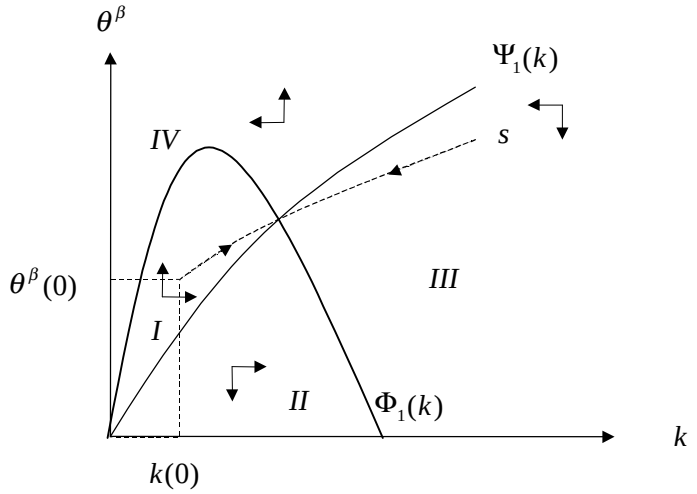

Figure 4: *Analysis of the Stability for the Steady-State.*

dynamic factor allocation function[17]

$$\theta^{\beta} = \frac{(1-\alpha)(1-\beta)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\left[\left(\frac{\alpha}{1+c_I}\right)k^{\alpha-1} + \beta\left(\hat{U}-\hat{V}\right) + \nu - \hat{\lambda}\right]}k^{\alpha}.$$

Considering $\hat{U} = \hat{V} = -\hat{\theta}$, the function can be rewritten as

$$\dot{\theta} = \frac{1}{\beta}\left[\frac{\alpha}{1+c_I}k^{\alpha-1} - \frac{(1-\alpha)(1-\beta)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}}k^{\alpha}\theta^{-\beta} + \nu - \hat{\lambda}\right]\theta. \tag{25}$$

Equation (25) shows the transitional dynamics for θ^{β}, i.e. labor market tightness increases if

$$\dot{\theta} > 0 \iff \theta^{\beta} > \frac{(1-\alpha)(1-\beta)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\left[\nu - \hat{\lambda} + \frac{\alpha}{1+c_I}k^{\alpha-1}\right]}k^{\alpha}.$$

Thus, labor market tightness increases, if the realized level of labor market tightness is greater than the equilibrium level and vice versa.

Furthermore, the following dynamic capital accumulation function can be derived as[18]

$$\dot{k} = \frac{s}{1+c_I s}\left[k^{\alpha} - \frac{c_{v0}\hat{\lambda}\left(\hat{E}+\nu\right)}{\lambda_0}\theta^{\beta}\right] - \left(\hat{\lambda}+\hat{E}\right)k. \tag{26}$$

[17] See appendix.

[18] For a detailed derivation see appendix.

Equation (26) shows the transitional dynamics for the capital intensity; it increases if

$$\dot{k} > 0 \iff \theta^{\beta} < \frac{\lambda_0}{c_{v0}\hat{\lambda}\left(\hat{E}+\nu\right)}\left\{k^{\alpha} - \left(\frac{1+c_I s}{s}\right)\left(\hat{\lambda}+\hat{E}\right)k\right\}.$$

Thus, if a capital intensity is realized lying below the balanced capital accumulation function, this capital intensity is too small to generate the equilibrium capital accumulation in labor efficiency units. The realized capital intensity has to increase in order to reach the equilibrium capital intensity and vice versa.

Due to this analysis, the transitional dynamics shown in Figure 4 are implied. To achieve the long-run steady-state the dynamic system not only has to be in the areas of I or III, it also has to be on the stable saddle path s and the starting variables $V(0), U(0), E(0)$ and $K(0)$ must have values such that they are already on the saddle path s in $t = 0$.

5 Economics of the Steady-State

For analyzing the effect of technical progress on long-term unemployment, its influence on the efficient factor allocation function, on the balanced capital accumulation function and on the equilibrium level of long-term unemployment for economies with a high respectively a low equilibrium level of capital intensity is derived in the following.

An increase in the rate of technical progress induces a change in the efficient factor allocation function and shifts the function downwards. Therefore, an inverse relation between efficient factor allocation and the growth rate of technical progress exists,[19] i.e.

$$\frac{\partial \Psi_1(k)}{\partial \hat{\lambda}} = -\frac{\overbrace{(1-\alpha)(1-\beta)(1-\omega)\lambda_0}^{>0}\overbrace{\left[\nu - 2\hat{\lambda} + \frac{\alpha}{1+c_I}k^{\alpha-1}\right]}^{>0}\overbrace{k^{\alpha}}^{>0}}{\underbrace{\hat{\lambda}^2\left[\nu - 2\hat{\lambda} + \frac{\alpha}{1+c_I}k^{\alpha-1}\right]^2}_{>0}} < 0.$$

For given capital intensity the increase in the growth rate of technical progress shifts the efficient factor allocation function downwards and reduced labor market tightness and increased unemployment is implied.

[19] For the derivation of the inverse relation see appendix.

Analyzing the effect of increasing productivity growth on the balanced accumulation function, a negative relationship can also be derived:

$$\frac{\partial \Phi_1(k)}{\partial \hat{\lambda}} = -\frac{\lambda_0 k^\alpha}{c_{v0}\nu \hat{\lambda}^2} < 0.$$

The increase in the growth rate of technical progress affects the balanced capital accumulation negatively and for given unemployment the labor market becomes tighter, i.e. labor market tightness decreases. This is reflected by the downward shift of the balanced accumulation function.

In order to examine the total effect of an increase of the rate of technical progress on steady-state labor market tightness and on steady-state unemployment, the above implications – both derived separately – have to be analyzed together for economies characterized by high or low capital intensities. Industrialized countries are economies with high levels of capital intensities, because they are innovation rather than imitation economies. This means that firms of those economies invent new technologies to realize cost advantages; and because they work with high capital intensities, they offer vacancies endowed with the latest technology. These firms require workers with human capital able to invent new production technologies and to handle the latest technologies. Therefore, they are innovators rather than imitators. On the other hand, economies with low capital intensities are imitation rather than innovation economies.

Depending on whether an economy has a high or low level of capital intensity, two effects influencing steady-state unemployment can be distinguished:[20] If the increase in the rate of technological progress implies that more existing jobs are destroyed than new ones are created, a reduction in aggregate employment appears and this is the creative destruction effect. However, a positive employment effect can results, which is the capitalization effect, if the rise in the growth rate of technological progress induces that more vacancies are produced than existing jobs are destroyed.

Innovation Economy

If an economy is an innovation country, the introduction of new technologies implies that old jobs become unproductive really fast causing the destruction of these unproductive jobs. Since quantitatively more old jobs are destroyed than new ones are created, the net dismissal of workers is implied and at a high equilibrium level of

[20] See also Mortensen, Pissarides (1998), Postel-Vinay (1998), Aghion, Howitt (1994) and Pissarides (1990).

capital intensity the creative destruction effect dominates the capitalization effect. Therefore, at a high equilibrium level of capital intensity increasing productivity growth induces a reduction in steady-state labor market tightness and an increase in the steady-state level of unemployment.

As the creative destruction effect dominates, it follows that through the introduction of new technologies jobs are destroyed. However, new jobs are also created in economies with high capital intensities and this leads to new vacancies in the labor market. The new vacancies are endowed with the latest technology and, due to a constant labor force, and, because of omitting on-the-job-search, they can be occupied by jobless workers only. Furthermore, since the unemployed are not trained and do not accumulate further human capital during their jobless time, they do not have the abilities and the know-how necessary for filling these vacant jobs. Because the unemployed cannot handle the latest technologies, they are not attractive for firms and less matching takes place. This implies a reduction in the steady-state level of matching and, since the reduction in matching is caused by lacking human capital, this effect is called the *qualification-mismatch effect*.

Bringing both effects – the creative destruction and the qualification-mismatch effect – in an economy with high capital intensity together, an increase in the rate of technical progress leads to lower steady-state labor market tightness (from $\widetilde{\theta}_0^{\beta}$ to $\widetilde{\theta}_1^{\beta}$, see Figure 5a) and for given vacancies an increase in steady-state unemployment is implied. However, it is not clear how the steady-state capital intensity is influenced by increasing technical progress – it can shrink or rise; both effects are possible.

Furthermore, as steady-state unemployment increases and steady-state labor market tightness decreases, i.e. for an average unemployed person it will be more difficult to become matched with a vacancy, since more jobless workers apply for the same vacancy, an average unemployed remains longer in the unemployment pool and steady-state duration of unemployment increases. Therefore, the reduction in steady-state labor market tightness induces that the matching probability for the unemployed shrinks and an increase in the steady-state duration of unemployment follows (from $\widetilde{\rho}_0$ to $\widetilde{\rho}_1$, see Figure 5b).

Considering the implications resulting from an increase in technological progress on steady-state long-term unemployment, two effects working in the same direction can be distinguished. First, the share of steady-state long-term unemployment rises simply because steady-state unemployment duration increases (from $\widetilde{\phi}_0$ to $\widetilde{\phi}_1$, see Figure 5c). Second, steady-state long-term unemployment additionally goes up,

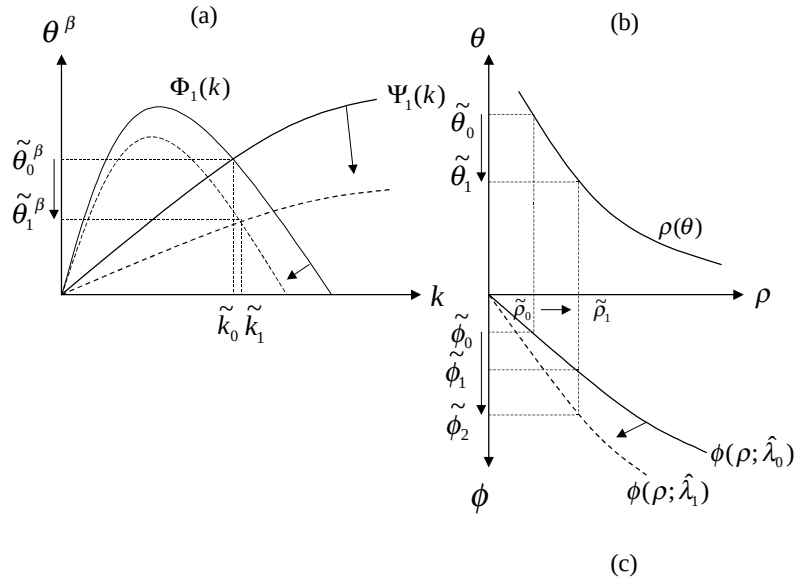

Figure 5: *Technical Progress and Long-Term Unemployment in an Innovation Economy.*

since the stock of difficult placeable jobless workers increases whereby this stock is characterized by the long-term unemployed itself. Since the introduction of technological progress generates vacancies endowed with the newest technology, firms demand only highly qualified workers. Because the unemployment pool is heterogeneous and consists of short-term and long-term unemployed, firms prefer to hire the short-term jobless workers. They possess most of their human capital and have better productive abilities than the long-term unemployed. Therefore, out of both unemployment groups, short-term unemployed are the potential candidates for the matching process since they do not have the stigma of human capital depreciation and of motivation losses. Due to this characteristics, job-matches are formed between firms and short-term unemployed implying that steady-state long-term unemployment increases additionally (from $\widetilde{\phi}_1$ to $\widetilde{\phi}_2$, see Figure 5c).

Imitation Economy

On the other hand regarding an economy with a low equilibrium level of capital intensity, the effect of an increase in technological progress on the labor market is ambiguous. In these economies the introduction of technical progress signifies that the production technologies being implemented are already known in economies with high capital intensities and, due to the import of goods and technologies from inno-

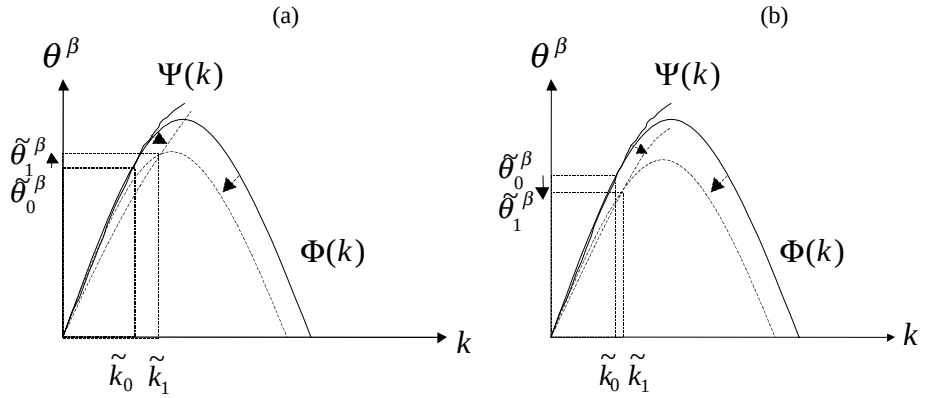

Figure 6: *Effects of Technical Progress on the Labor Market in an Imitation Economy.*

vation countries, they are also known in imitation countries. This reflects the idea of the importance of the stage of development and of the catching-up processes of economies with low capital intensities. In such an economy, the stage of development is that of an imitator and the economy has not reached the innovation stage yet. Due to increasing technological progress, firms create new vacancies and offer them in the labor market and, due to the import of production methods, the technologies as well as the handling of the technologies are well-known in these economies. Thus, unemployed workers are available possessing the human capital to fill the vacancies and additional matching takes place. This is the capitalization effect that increases employment and decreases unemployment.

However, the creative destruction effect is also present in economies with low capital intensities implying that old unproductive jobs are destroyed, that workers are laid off and that unemployment increases.

Because the creative destruction effect or the capitalization effect can dominate, the implication for steady-state labor market tightness is ambiguous. As long as the capitalization effect dominates, steady-state labor market tightness will increase (see Figure 6a) inducing a reduction in steady-state unemployment, in steady-state unemployment duration and in the fraction of steady-state long-term unemployment. However, if the creative destruction effect dominates, the effects are the opposite (see Figure 6b).

Thus, for an economy with a low equilibrium level of capital intensity, increasing technological progress can have ambiguous effects for the steady-state labor mar-

ket depending on the dominance of one of these effects and, therefore, steady-state duration as well as steady-state long-term unemployment can shrink or rise.

6 Conclusion

The starting point of this analysis is the stylized fact that in industrialized countries characterized by high levels of capital intensities and rapidly introducing technical progress long-term unemployment increases and simultaneously the countries display positive GNP per capita growth rates.

For explaining this stylized fact, the relationship between long-term unemployment and the rate of technical progress is analyzed in a growth-matching-model that describes labor market frictions and the capital accumulation process of an economy. The unemployment pool consists of heterogeneous unemployed workers and the fraction of long-term unemployment is endogenously determined by the duration of unemployment itself and by the rate of technical progress.
Since industrialized countries are economies with high levels of capital intensities, they are innovation rather than imitation economies and, due to inventing rapidly new production methods, they offer vacancies endowed with the latest technology. Therefore, firms require workers with human capital able to handle the most recent technologies. Assuming a constant labor force and a heterogeneous unemployment pool that consists of short-term and long-term unemployed, only jobless workers can fill new vacancies. Furthermore, since the qualification level of the unemployed does not grow with the same rate as technical progress, the human capital of the unemployed depreciates as technical progress increases and it depreciates the faster, the longer the unemployment duration takes. Firms are not willing to hire long-term unemployed implying that only short-term unemployed are matched with new vacancies. Thus, the model shows that in innovation economies with high steady-state capital intensities qualification-mismatch increases by accelerating technical progress and, due to the dominance of the creative destruction effect and the qualification-mismatch effect over the capitalization effect, long-term unemployment rises as well.

For imitation economies with low steady-state capital intensities, increasing technological progress can be favorable or less favorable for long-term unemployment depending on whether the creative destruction effect or the capitalization effect dominates.

7 Appendix

Lemma 1 *Using (12) and (13), then* $F_K(k) = (1 + c_I)\, r$.

Proof. Differentiate (12) w.r.t. time and substitute it in (13), then $F_K(k) = (1 + c_I)\, r$ is implied. ■

Lemma 2 *Using (2), (9), (10),* $c_v = c_{v0} e^{\hat{\lambda} t}$ *and* $\lambda = \lambda_0 e^{\hat{\lambda} t}$, *then* $F_E(k) = w + \frac{c_v(t)\hat{\lambda}}{1-\beta}\theta^{\beta}\left[r - \hat{\lambda} + \beta\left(\hat{U} - \hat{V}\right) + \nu\right]$.

Proof. Differentiate (2) w.r.t. V and (9) w.r.t. time, use $c_v := c_{v0} e^{\hat{\lambda} t}$ and $\lambda := \lambda_0 e^{\hat{\lambda} t}$, then

$$\begin{aligned} & e^{-rt} c_v \left(r - \hat{\lambda}\right) + \frac{1-\beta}{\hat{\lambda}}\left[\dot{\mu}_1 + \mu_1 \beta\left(\hat{U} - \hat{V}\right)\right]\theta^{-\beta} = 0 \\ \Leftrightarrow \quad & -\dot{\mu}_1 = \frac{\hat{\lambda}}{1-\beta} e^{-rt} c_v \left(r - \hat{\lambda}\right)\theta^{\beta} + \mu_1 \beta\left(\hat{U} - \hat{V}\right). \end{aligned}$$

Substitute (10) for $-\dot{\mu}_1$, then

$$\frac{\hat{\lambda}}{1-\beta} e^{-rt} c_v \left(r - \hat{\lambda}\right)\theta^{\beta} + \mu_1 \beta\left(\hat{U} - \hat{V}\right) = e^{-rt}\left[F_E(k) - w\right] - \mu_1 \nu$$

and substitute (9) for μ_1, then

$$\begin{aligned} & c_v \frac{\hat{\lambda}}{1-\beta}\theta^{\beta}\left[\beta\left(\hat{U} - \hat{V}\right) + \nu\right] = F_E(k) - w - c_v \frac{\hat{\lambda}}{1-\beta}\theta^{\beta}\left(r - \hat{\lambda}\right) \\ \Leftrightarrow \quad & F_E(k) = w + c_v \frac{\hat{\lambda}}{1-\beta}\left[r - \hat{\lambda} + \beta\left(\hat{U} - \hat{V}\right) + \nu\right]\theta^{\beta}. \end{aligned}$$

Therefore, (16) is implied. ■

Proposition 3 *Using (1), (6), (7a), (15), (16) and (20), then the efficient factor allocation function* $\Psi_1(k) := \frac{(1-\alpha)(1-\beta)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\left[\left(\frac{\alpha}{1+c_I}\right)k^{\alpha-1} + \nu - \hat{\lambda}\right]} k^{\alpha} = \theta^{\beta}$ *follows.*

Proof. Differentiate (7a) w.r.t. E, substitute this and (6) in (16), then

$$\theta^{\beta} = \frac{(1-\alpha)(1-\beta)(1-\omega)\lambda}{c_v \hat{\lambda}\left[r + \beta\left(\hat{U} - \hat{V}\right) + \nu - \hat{\lambda}\right]} k^{\alpha}.$$

Differentiate (7a) w.r.t. K and substitute this, (15), $\lambda = \lambda_0 e^{\hat{\lambda} t}$ and $c_v = c_{v0} e^{\hat{\lambda} t}$ in the above equation, then

$$\theta^{\beta} = \frac{(1-\alpha)(1-\beta)(1-\omega)\lambda_0}{c_{v0}\hat{\lambda}\left[\frac{\alpha}{1+c_I} k^{\alpha-1} + \beta\left(\hat{U} - \hat{V}\right) + \nu - \hat{\lambda}\right]} k^{\alpha}.$$

Furthermore, differentiate (1) w.r.t. time, then $\dot{E} = -\dot{U}$, use $\theta := V/U$, then $\dot{\theta} = \dot{V}/U - V\dot{U}/U^2$; use (1) and (20), then $\dot{E} = 0 = -\dot{U}$, $\dot{\theta} = 0$ and $\dot{V} = 0$ are implied and therefore $\hat{U} = \hat{V} = 0$. Substitute this in the above equation, the efficient factor allocation function $\Psi_1(k)$ follows. ■

Proposition 4 *Suppose* $\nu - \hat{\lambda} > 0$ *and* $k > (a_2/a_3)^{1/1-\alpha}$, $\Psi_1(k)$ *is an increasing concave function with* $\Psi_1(0) = 0, \Psi_1(\infty) = \infty, \Psi_1'(0) = a_1/a_2 < \infty$, $\Psi_1'(\infty) = 0$, $\Psi_1'(k) > 0, \Psi_1''(k) < 0$.

Proof. Equation (21) is equivalent to

$$\begin{aligned} & \Psi_1(k) = \frac{a_1 k^{\alpha}}{a_2 k^{\alpha-1} + a_3} \\ \Leftrightarrow \quad & \Psi_1(k) = \frac{a_1 k}{a_2 + a_3 k^{1-\alpha}} \end{aligned}$$

with $a_1 := \left[\lambda_0 (1-\alpha)(1-\omega)(1-\beta)/c_{v0}\hat{\lambda}\right]$, $a_2 := \alpha/(1+c_I)$ and $a_3 := \nu - \hat{\lambda}$, then

$$\begin{aligned} \Psi_1(0) &= 0, \\ \Psi_1(\infty) &= \infty. \end{aligned}$$

Using $\nu - \hat{\lambda} > 0$, the properties of $\Psi_1'(k)$ follow directly from

$$\Psi_1'(k) = \frac{a_1\left[a_2 + a_3 k^{1-\alpha}\right] - (1-\alpha) a_1 a_3 k^{1-\alpha}}{\left[a_2 + a_3 k^{1-\alpha}\right]^2},$$

then

$$\begin{aligned} \Psi_1'(k) &= \frac{a_1 a_2 + \alpha a_1 a_3 k^{1-\alpha}}{\left[a_2 + a_3 k^{1-\alpha}\right]^2} > 0, \\ \Psi_1'(0) &= \frac{a_1}{a_2} < \infty, \\ \lim_{k\to\infty} \Psi_1'(\infty) &= \lim_{k\to\infty} \frac{\alpha a_1}{2\left[a_2 + a_3 k^{1-\alpha}\right]} = 0. \end{aligned}$$

Furthermore using $k > (a_2/a_3)^{1/1-\alpha}$, the properties of $\Psi_1''(k)$ follow directly from

$$\begin{aligned} \Psi_1''(k) = & \underbrace{-2a_1 a_2\left[a_2 + a_3 k^{1-\alpha}\right]^{-3}(1-\alpha) a_3 k^{-\alpha}}_{<0} \\ & + \underbrace{\alpha a_1 a_3 k^{1-\alpha}\left[a_2 + a_3 k^{1-\alpha}\right]^{-2}(1-\alpha)}_{>0} \underbrace{\left\{1 - \frac{2a_3 k^{1-\alpha}}{a_2 + a_3 k^{1-\alpha}}\right\}}_{<0}, \end{aligned}$$

then

$$\Psi_1^{''}(k) < 0$$

is implied. ■

Proposition 5 *Using (1), (3) and (20), the steady-state employment rate* $e(\theta) = \frac{p(\theta)}{\nu + p(\theta)}$ *is implied.*

Proof. Equation (20) can be written as

$$\dot{E} = 0 \Leftrightarrow M = \nu E$$

and using (1) and (3), then

$$\begin{aligned} & \frac{M}{U} U = \nu E \\ \Leftrightarrow \quad & \frac{M}{U} \overline{L} - \frac{M}{U} E = \nu E \\ \Leftrightarrow \quad & p(\theta) \overline{L} = (\nu + p(\theta))\, E. \end{aligned}$$

Therefore, $e(\theta) := \frac{E}{\overline{L}} = \frac{p(\theta)}{\nu + p(\theta)}$ follows. ■

Proposition 6 *Using (2), (7b), (8), (18), (19) and (23), the balanced accumulation function* $\theta^{\beta} = \frac{\lambda_0}{c_{v0} \hat{\lambda} \nu} \left[k^{\alpha} - \frac{(1 + c_I s) \hat{\lambda}}{s} k \right] =: \Phi_1(k)$ *is implied.*

Proof. Using equations (18), (19) in efficiency units, then

$$x = (1 + c_I s)\, y + c_v v. \tag{27}$$

Define $v := V / \lambda E$ and use (2) and (8), then

$$v = \frac{\left(\hat{E} + \nu \right) \hat{\lambda}}{\lambda} \theta^{\beta}. \tag{28}$$

Substituting (7b) and (28) in (27), then

$$y = \frac{1}{1 + c_I s} \left[k^{\alpha} - \frac{c_{v0} \hat{\lambda} \left(\hat{E} + \nu \right)}{\lambda_0} \theta^{\beta} \right].$$

Define $k := K / [\lambda E]$, then $\dot{k} = \dot{K} / \lambda E - \left(\hat{\lambda} + \hat{E} \right) k$. Use $\dot{K} = I = sY$, i.e. use (19), then

$$y = \frac{1}{s} \left[\left(\hat{\lambda} + \hat{E} \right) k + \dot{k} \right].$$

Equate the last equations and use (23) and $\dot{E} = \hat{E} = 0$, then

$$\theta^{\beta} = \frac{\lambda_0}{c_{v0}\hat{\lambda}\nu}\left[k^{\alpha} - \frac{(1+c_I s)\,\hat{\lambda}}{s}k\right] =: \Phi_1(k)$$

is implied. ■

Proposition 7 *The balanced accumulation function is a concave function with* $\Phi_1(0) = 0$, $\Phi_1(\infty) = -\infty$,$\Phi_1'(0) = \infty$, $\Phi_1'(\infty) = -a_5$,$\Phi_1'(k) \begin{cases} \geq 0 \\ < 0 \end{cases} \quad for \quad \alpha k^{\alpha-1} \begin{cases} \geq a_5 \\ < a_5 \end{cases}$ *and* $\Phi_1''(k) < 0$.

Proof. Rewrite (24) as $\Phi_1(k) = a_4\left[k^{\alpha} + a_5 k\right]$ with $a_4 := \lambda_0/c_{v0}\hat{\lambda}\nu$ and $a_5 := (1+c_I s)\,\hat{\lambda}/s$, then $\Phi_1(0) = 0$, $\Phi_1(\infty) = -\infty$. Differentiate $\Phi_1(k)$ w.r.t. k, then

$$\Phi_1'(k) \begin{cases} \geq 0 \\ < 0 \end{cases} \quad for \quad \alpha k^{\alpha-1} \begin{cases} \geq a_5 \\ < a_5 \end{cases}$$

and therefore

$$\begin{aligned} \lim_{k\to 0}\Phi_1'(k) &= \lim_{k\to 0}\left(\alpha k^{\alpha-1} - a_5\right) = \infty \\ \lim_{k\to\infty}\Phi_1'(k) &= \lim_{k\to\infty}\left(\alpha k^{\alpha-1} - a_5\right) = -a_5. \end{aligned}$$

Furthermore,

$$\Phi_1''(k) = (\alpha-1)\,\alpha a_4 k^{\alpha-2} < 0$$

is implied. ■

Proposition 8 *Suppose* $\nu - 2\hat{\lambda} > 0$, *then* $\frac{\partial\Psi_1(k)}{\partial\hat{\lambda}} < 0$.

Proof. Take the derivative of (21) w.r.t. $\hat{\lambda}$ and use $\nu - 2\hat{\lambda} > 0$, then

$$\frac{\partial\Psi_1(k)}{\partial\hat{\lambda}} = -\frac{(1-\alpha)(1-\beta)(1-\omega)\lambda_0\left[\nu - 2\hat{\lambda} + \frac{\alpha}{1+c_I}k^{\alpha-1}\right]k^{\alpha}}{c_{v0}\hat{\lambda}^2\left[\nu - \hat{\lambda} + \frac{\alpha}{1+c_I}k^{\alpha-1}\right]^2} < 0$$

is implied. ■

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